PROXY MODEL FOR RESERVOIR SIMULATION

BY

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ABSTRACT

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Reservoir management decisions have the greatest impact on the successes of the business in the oil industry. As the importance of reservoir decisions increase, so the importance of information that these decisions are based on. Information should be available at any time needed with the highest level of accuracy. The available reservoir management tools vary between numerical reservoir simulation models that have high accuracy but lack the speed in providing the required results, even though huge advancements were achieved in the computational resources, and simple models that give results in a fraction of a second but fail in providing the desired level of accuracy.

Over the years, researchers and reservoir engineers tried to come up with different reservoir models that can combine both speed and accuracy. A good model can be a powerful reservoir management tool and therefore should be capable to perform analysis for any given characteristic, match the past performance and predict the future.

In this study a new proxy model is proposed as a computationally inexpensive alternative for numerical reservoir simulation. The model includes reservoir static and dynamic parameters represented in rock properties, fluid properties and operational conditions. The proposed model will be evaluated and compared with other methods in the aspects of speed, accuracy and computational cost. The result of this study is a simple reservoir model that can describe the reservoir performance at any specified point in time with a good level of accuracy in a fraction of a second, and can be used in probabilistic forecasting, history matching, sensitivity analysis and developmental optimization.

ملخص الرسالة

الاسم الكامل: محمد عبد الوهاب عبد الله صالح

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لقرارات إدارة المكتمن الأثر الأكبر على نجاح الأعمال في صناعة النفط. ومع ازدياد أهمية قرارات المكامن، فإن المعلومات التي تستند إليها هذه القرارات تزداد اهمية. وينبغي أن تكون المعلومات متاحة في أي وقت لازم مع أعلى مستوى من الدقة. وتختلف أدوات إدارة المكامن المتاحة بين نماذج المحاكاة العددية للمكامن ذات الدقة العالية ولكنها تفتقر إلى السرعة في توفير النتائج المطلوبة، على الرغم من تحقيق تقدم هائل في الموارد الحسابية، ونماذج بسيطة تعطي نتائج في جزء من الثانية ولكنها تفشل في توفير المستوى المطلوب من الدقة.

على مر السنين، حاول الباحثون ومهندسي المكامن التوصل إلى نماذج المكامن التي يمكن أن تجمع بين السرعة ومستويات عالية من الدقة، واقترح العديد من النماذج من أجل تحقيق ذلك. يجب أن يكون النموذج الجيد الذي يمكن أن يكون أداة قوية لإدارة المكمن قادرا على إجراء التحليل لأي خاصية معينة، والتنبؤ في المستقبل وكذلك مطابقة الأداء الماضي.

في هذه الدراسة يقترح نموذج جديد كبديل حسابي غير مكلف للمحاكاة العددية للمكامن. ويشمل النموذج خواص المكامن الثابتة والديناميكية الممثلة في خصائص الصخور، وخصائص السوائل والظروف التشغيلية. وسيتم تقييم النموذج المقترح ومقارنته بطرائق أخرى في جوانب السرعة والدقة والتكلفة الحسابية. وكانت نتيجة هذه الدراسة نموذج مكمن بسيط يمكن أن يصف أداء المكمن في أي نقطة زمنية محددة بمستوى جيد من الدقة في جزء من الثانية، ويمكن استخدامه في التنبؤ الاحتمالي ومطابقة التاريخ وتحليل الحساسية والتطوير.

CHAPTER 1

INTRODUCTION

Hydrocarbon reservoirs are large volumes of porous rock containing hydrocarbons at various depths from the surface of the ground. Reservoir engineering is the application of scientific principles to the fluid flow problems during the development and production of hydrocarbon reservoirs to obtain an economic recovery. Tools of the reservoir engineer vary between subsurface geology, applied mathematics, and the laws of physics and chemistry governing the behavior of fluids in reservoir rock. Reservoir simulation is a part of reservoir engineering that can be defined as the use of computer models to describe the flow of fluids in porous rock. The objective of reservoir simulation is to mimic the reservoir performance in order to study how the reservoir will act under certain operation conditions to maximize the recovery from these reservoirs.

Reservoir simulation has become the prediction tool for the industry. It can predict accurate performance for a reservoir under various conditions. In the oil industry every project has a huge investment that is accompanied by many risks, which has to be minimized by selecting the best scenarios for production and development. This risk arises from the complexity of many factors such as the reservoir rock and the fluids contained in it. These complexities can be overcome by adding it to the input data in the simulation model using sound engineering practices and accurate reservoir simulation.

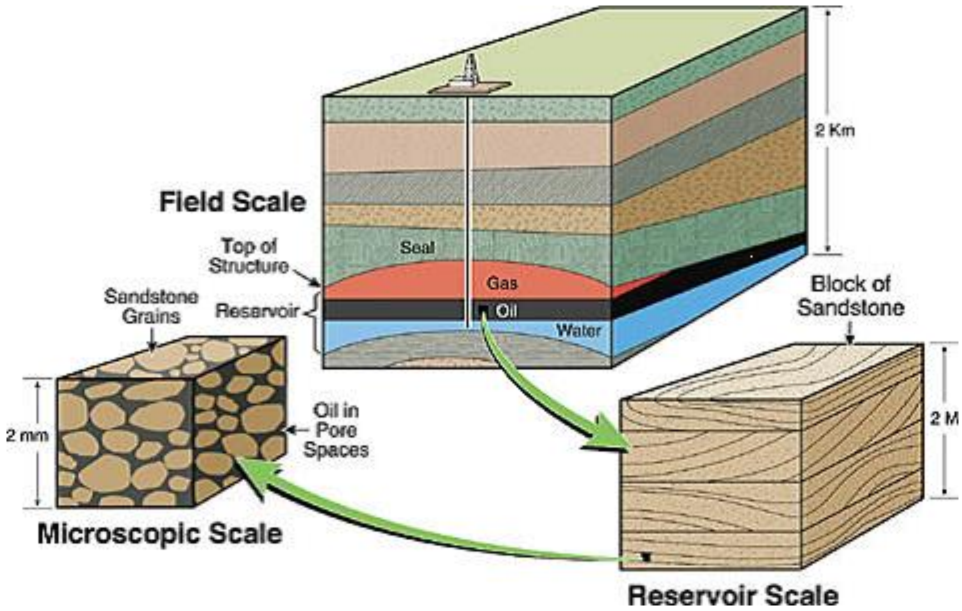

Figure 1.1: Hydrocarbon Reservoirs (Zitha et al. 2008)

The reservoir model is constructed through four connected stages: creation of the physical model, developing a mathematical model, discretization of the model and finally creating an algorithm for the solution. Therefore, in order to perform reservoir simulation, the skills of a physicist, mathematician, reservoir engineer and a computer scientist, all are required.

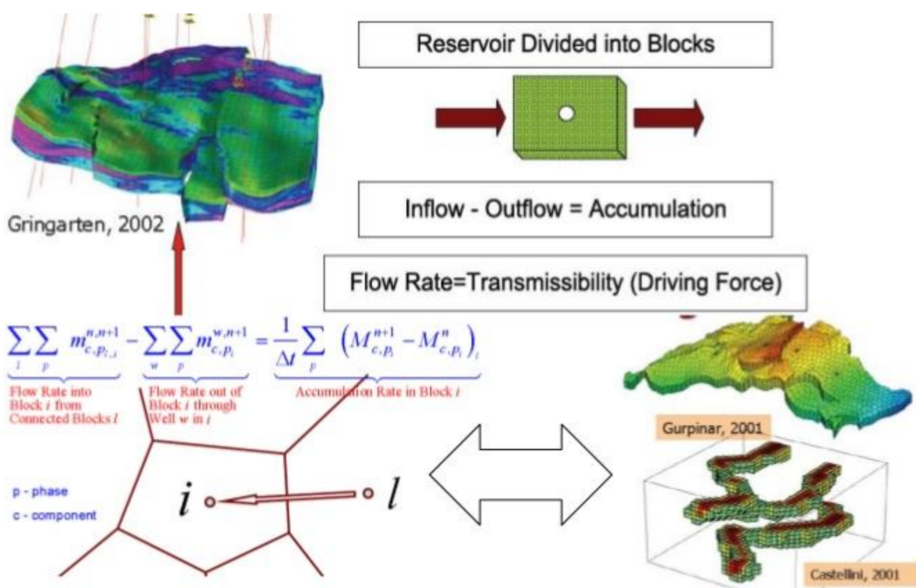

Figure 1.2: Reservoir Modeling Process

Although reservoir simulation is not new, what new is the more detailed reservoir characteristics, resulting in more accurate simulations, which have become practical because of the advanced computational resources available. However, the more detailed simulation requires more complex mathematical expressions, and these complex expressions are more difficult to understand.

Proxy models are a mathematical or statistical replication of a simulation model outputs for specific inputs. The terms response surface model, surrogate model, meta-model is also used to refer to proxy models. However, the term Proxy models is more commonly used in the oil industry. They are applicable for many different areas of science as approximation for numerical modeling. Application areas for proxy models include:

- Sensitivity analysis
- Probabilistic forecasting
- History matching
- Development planning and optimization

Combined with experimental techniques, proxy models are used for sensitivity analysis. Using one-parameter-at-a-time approach in linear sensitivity and advanced experimental design for correlations and higher order effects. Proxy models are also used for sampling in Monte Carlo simulation in order to perform probabilistic forecasting.

History matching process requires many simulation runs to explore uncertainties and find an acceptable solution. Proxy models can represent the simulation outputs, so they fit adequately for history matching. Same as history matching is development planning and optimization, it may require even more simulation runs which can become problematic even in the existence of computational clusters. This is the reason why proxy models are used to asset production optimization.

Creating a proxy model with a high quality depends on the quality of the inputs. Reservoir simulation deals with highly non-linear outputs. Therefore, inputs uniformly distributed along the uncertainty space are not sufficient to construct an adequate proxy model.

A proxy model is a replica for traditional numerical reservoir simulation model. It may be questioned that why do we need a replication when a numerical reservoir simulation model exists. Necessity of proxy models comes from the fact that they are simple and robust models, while it takes a long period of time for a single run in numerical reservoir simulation. As the model becomes more complex it requires considerable longer run-time due to the amount of calculations and iterations even on advance computational configurations. Exhaustive and comprehensive evaluation of the solution space for designing field development strategies as well as quantification of uncertainties associated with the static model are the type of analyses that require large number of simulation runs. Where a numerical simulation takes hours for a single run, or requires a very expensive computational resources, this analysis become impractical and it has to be compromised by designing and running a much smaller number of runs to help in making a decision.

CHAPTER 2

LITERATURE REVIEW

2.1 Reservoir Modeling

Reservoir modeling has been a major research subject in reservoir engineering. There is a vast amount of papers available and a significant number of ongoing studies; a brief review is given here to summarize the related works. (Weber et al. 1990) presented some practical methods to eliminate jargon in reservoir modeling by presenting a simple system that links between geology and fluid flow concepts. This system will improve gridding of the reservoir by considering the reservoir architecture.

Years later,(Branets et al. 2009) gave a look at the challenges facing reservoir modeling and the technologies that can be used to overcome those challenges. They combined advanced gridding techniques with accurate global scale-up method to construct coarse simulation models. Also they stated that reservoir descriptions becoming more complex because of the high resolution in remote sensing technologies, the huge amount of production data from the field, and better geological understanding.

As the oil industry is growing, new resources were introduced (e.g. shale gas, gas hydrates) that came up as a new challenge for reservoir modeling. Now the area of research became vast and many studies were presented targeting the new challenges. (Cipolla et al. 2010) worked in the area of shale gas reservoirs, and demonstrated the effect of gas desorption on the production profile and the ultimate recovery from the reservoir. There outcomes can be summarized as: gas desorption has no great impact on moderate-to deep shale gas reservoirs, and the desorbed gas only represent 5-15% of the ultimate recovery from the reservoir that can be produced in the later life of the field.

(Kuchuk, Biryukov, and Fitzpatrick 2014) presented a model for fractured reservoirs and examined it from microscopic and macroscopic representative elementary volume REV point of view. Their work showed that if the fractures permeability is larger than the matrix permeability by some orders of magnitude, the macroscopic REV can be constructed and dual porosity model can be used. If the difference in permeability is huge, then macroscopic REV cannot be constructed and analytical or numerical techniques should be used.

As the models become more complex and incorporate different parameters, the process of reservoir simulation becomes more expensive in the aspects of computational resources needed to perform the simulation and the process running time. At the same time reservoir management needs accurate fast information, business management needs to reduce the amount of investment and increase their revenues; therefore, long simulation runs and high cost computational resources represent an issue that should be taken into consideration.

2.2 Proxy Models

In recent years, researchers were trying to come up with simple reservoir models that can substitute full numerical reservoir simulation while maintaining high levels of accuracy. Many studies were published presenting what is now called reduced models. These reduced models or proxy models can be divided into:

- Upscaling Models
- Reduced Order Models
- Artificial Intelligence Models

2.2.1 Upscaling Models

Upscaling, sometimes called homogenization, refers to the process of replacing number of grids in the reservoir with a degree of heterogeneity, with respect of a certain parameter (i.e. permeability), with one grid or a less number of grids with a single representative value for the specific property.

Many authors used reservoir upscaling in order to reduce the number of the grids in reservoir simulation; which leads to speed up the simulation process. (Christie 1996) stated that the necessity of upscaling is bridging the gap between detailed reservoir descriptions and an upscale algorithm that gives suitable values for flow functions such as porosity and permeability. It was found that single phase upscaling is the best understood form of upscaling in the calculation of effective permeability.

(Stanford 1998) proposed an upscaling method based on a semi-analytical simulator. A coarse grid is generated based on stream tubes and isobars upscale properties in two or three dimensions. The same methodology is used to generate relative permeability curves without requiring any additional time. The main idea is that a stream tube is a line that connects an injector with a producer. They stated that this method can be integrated to give a solution for the large and complex reservoir models, as the technique used can adapt to any change in the stream tube geometry due to a change in the reservoir condition.

(Ali, Al-qassab, and Aramco 2000) performed a process that integrates all possible data available (logging, RFT, Production history and geological interpretations) in a geological static model, this model is used to create a dynamic reservoir simulation model. The resultant model was up scaled using a multi-step technique that accounts for all the varying details in a complex reservoir. The result is that a match of pressure and water saturation was obtained and then prediction cases were developed for decision making and developmental planning.

The CPU time of the dynamic modeling was reduced due to the knowledge of faults that guided the model of the transmissibility modifications. Also the history match period is reduced by half and CPU per time step was optimized using the new technique.

2.2.2 Reduced Order Models

(Cardoso 2009) stated that reduced-order modeling procedures may be very useful for optimization problems. In their work they described a reduced-order technique named the trajectory piecewise linear (TPWL) procedure for water flooding optimization. They achieved a high degree of efficiency because of the reduction in the computations required. This work showed that reduced-order models can be used for determining optimal well settings.

Some authors used reduced models to describe certain process, (Ghasemi and Whitson 2011) used an isothermal black-oil simulator to model steam-assisted gravity drainage (SAGD). SAGD design requires many simulation runs to find operational conditions that maximize economic value – e.g. injector and producer location, rates, pattern spacing, and steam chamber temperature. The proposed black-oil proxy model runs up to 10 times faster than a thermal model, while maintaining similar performance behavior. The run time comparison between the SAGD process simulation using a thermal model and the black-oil proxy equivalent was a CPU time ratio of ~8.

(Artus, Tauzin, and Houzé 2014) investigated the performance of four reduced-order numerical proxies to replace the traditional detailed one to simulate unconventional resources. Models gave acceptable results for simple geometries, and because of the short simulation time that is independent from the number of fractures they represent a good tool for history matching. Some proxies were based on grid manipulation, so by reducing the size of the system they reduced the simulation time by 20-50% while maintain good accuracy. The models accelerated the history matching process and the uncertainty analysis.

(Fillacier et al. 2014) used production history to quantify uncertainties in the future by utilizing two related and complementary mechanisms. The first method is to create ensemble of simulation runs by taking a moderately small sample of parameter combinations, and the second is by creating a proxy model to calculate the prediction responses of interest. Both methods produced consistent results when applied to a standard test case, and the produced uncertainty is compatible with theoretical values. They also showed that proxy modeling can successfully be based on ensemble simulation runs, and the combination of the two methods provides a coherent approach to quantify uncertainty prediction.

Another work on history matching and uncertainty prediction, (He et al. 2016) proposed the proxy-for-data approach, where one proxy is constructed for each observation data point then use the values predicted by these proxies to calculate the aggregated mismatch. Because proxies are constructed for the data themselves rather than for the aggregated mismatch, the nonlinearity of the aggregated mismatch definition will not affect the quality of the proxy. The new approach was successfully applied to both synthetic and field examples and showed improved proxy quality for both cases.

(Zhao, Kang, and Exploration 2016a) derived and implemented an interwell-numerical-simulation model (INSIM) to model the performance of a reservoir under water flooding. In this model the reservoir is characterized as coarse model with number of interwell control units, where each unit has two parameters: transmissibility and control pore volume. The model parameters estimated from history matching provide a relative characterization of interwell-formation properties. The well interactions are assumed to be fixed. Using INSIM it is possible to calculate the oil and water flow rates and hence history match water-cut data. This method can be used for water flooding optimization but with far-less computational effort than with the traditional method by use of a reservoir simulator.

2.2.3 Artificial Intelligence Models

Many authors proposed reduced models based on different Artificial Intelligent approaches like neural networks, genetic algorithms and radial basis functions. The proposed models were used for different purposes, sometimes as a replacement for full numerical simulation other times as a simulator for a specific process.

(Yu, Wilkinson, and Castellini 2008) using Genetic Programming GP, they constructed proxies for full reservoir simulation to replace the high cost simulators in order to sample larger number of reservoir models to get more information leading to a better decisions. Although the production data were very noisy and a significant production history was available, the proxy results were matching with the results given by the numerical full reservoir simulator. (Kalantari-Dahaghi, Esmaili, and Mohaghegh 2012) developed, calibrated and validated a Surrogate model for shale gas reservoirs that is based on the artificial intelligence. It was found that the surrogate model mimics the numerical simulation accurately, and it gives accurate results in a fraction of a second so it serves reservoir management effectively. Two years later (Mohaghegh and Abdulla 2014) used history matched reservoir model to increase the usage of numerical proxy model based on the technique of artificial intelligent. Surrogate Reservoir Models SRM summarize the complexity of the numerical model and copy its behavior, thus provide tools that help in making a decision.

(Amini et al. 2014) developed a grid based surrogate reservoir model to replace complex reservoir models. The model can give results for the pressure and the saturation on the scale of grid blocks in seconds. The characteristics of artificial intelligence allow the use of several types of input to train the model and give it flexibility to adjust when the inputs change within a specified range. Due to the large amount of data in the grid level, there should be a sampling method for the training data selection taking in consideration that the selected data represent the reservoir.

AI models were used for different processes, as example (Ghassemzadeh and Hashempour 2016) proposed a new approach to optimize gas lift system using proxy model to minimize an objective function. They coupled a machine learning based proxy model with a genetic algorithm and execute it over many time steps to give real time optimization. The model consists of the appropriate reservoir and well parameters to be able to imitate the relation between the inputs and the outputs, and so create a powerful tool to optimize the production system.

Another application is the optimization of production in a giant mature field in United Arab Emirates. (Solutions and Adco 2015) developed a surrogate reservoir model SRM, which is a smart proxy of numerical simulation, to address the following short comings of traditional reservoir simulation and reduced proxy models:

- Numerical reservoir simulation provides required accuracy, but takes longer time for simulation run so it is not suitable for processes that require many simulation runs like optimization.
- Conventional proxy or reduced models give results in a short period of time, but lack the required accuracy so it is not suitable for processes that require accurate results like history matching.

Different methodologies were used to come up with a better representation for Hydrocarbon reservoirs. Each methodology has its pros and cons, but all of them have the same objective, which is creating an accurate inexpensive alternative for numerical reservoir simulation. Some authors compared different approaches to see which one can give a better result. (Crick 2010) compared different proxy modeling techniques to check for their applicability to replace full reservoir simulation in history matching, production optimization and forecasting. The models techniques under study were:

- Polynomial regression
- Kriging
- Thin plate spline model
- Artificial Neural Network

All the models were found to be dependent on the complexity of the model, dimension of design space and input data. The most successful usage of proxy models was in the prediction of hydrocarbon initially in place HCIIP and oil recovery.

It was recommended that, taking any decision based on the results obtained by the proxy models requires the understanding of their limitations, and a quality assurance process to quantify the errors.

2.3 Differential Evolution

Differential evolution (DE) is arguably one of the most powerful stochastic real-parameter optimization algorithms in current use. DE is a population based Evolutionary Algorithm, it is an improvement of Genetic Algorithms. Simple GA uses a binary coding for representing problem parameters whereas DE uses real coding of floating point numbers. DE is used for minimizing possibly nonlinear and non-differentiable continuous space functions, it converges faster and with more certainty than many other acclaimed global optimization methods. Some advantages of this method that it requires few control variables, is robust, easy to use, and lends itself very well to parallel computation.

(Storn and Price 1997) firstly introduced DE to minimize continuous space functions. It was stated that DE is very simple, robust and straight forward technique that requires only few control variables, which are easy to be obtained from well-defined numerical interval.

For solving practical applications quickly using DE it is important to have the knowledge of choosing the most suitable control variables for a certain problem.

After Storn and Price, many researchers worked on the development of DE. (Babu and Jehan 2003) used DE to solve two test problems one on Multi-objective optimization and the second on classical Himmelhlau function, simulations were conducted involved solving:

- both problems using Penalty function method
- first problem using Weighing factor method and finding Pareto optimum set for the chosen problem

Compared to simple GA, DE gave the exact optimum solution with less number of iterations, which means faster and accurate method of optimization.

One of the best properties of DE is the applicability of modification in the algorithm in order to speed up the process of identifying the optimum. (Kaelo and Ali 2006) conducted numerical studies on 50 test sets from practical applications using a modified DE algorithms. It was found that the new DE is far superior to the traditional one even in the accuracy of identifying the global minimum. As mentioned, DE can be modified in order to fit a specific problem and improve the accuracy of obtaining the solution. (Babu and Angira 2006) introduced a modification that enhances the convergence rate without compromising on solution quality to handle non-differentiable, non-linear and multimodal objective function chemical engineering problems. The results clearly showed the improvement in the performance of DE with regard to the number of function evaluations (NFE)/CPUtime required to find the global optimum.

(Das and Suganthan 2011) presented an overal picture of the state of the art based on researches conducted on DE. Eventhough a single cure for all optimization problems does not exit, researchers over the years worked on DE by changing and manipulating the various consistuents of the algorithim, yet DE showed a great performance in the optimization of a wide range of multi-dimensional, multiobjective and multimodal optimization problems. In their study they gave an overview of the different most significant engigineering applications of DE and the possible directions of future research.

CHAPTER 3

RESEARCH OBJECTIVES AND METHODOLOGY

3.1 Research Motivation

Latest improvement and advancement in the computational hardware and software gave reservoir modeling a huge step forward, as now more complex systems can be modeled and put under study. The problem is the availability of these resources is limited or the access to it is restricted, and even if it is available they represent a very expensive asset to purchase, which increase the amount of the investment which leads to the increase in the size of any risk that used to be very small. If the computational resources were found in an affordable pricing and open access for anyone, there still another problem that the numerical reservoir simulation models take long periods of time to perform a single run especially when the system is very complex, such as fractured unconventional reservoirs. This will be an obstacle in any process that requires many simulation runs like development optimization.

A proxy model is an inexpensive alternative for numerical reservoir simulation, it has the capability of reproducing highly accurate well-based simulation responses as a function of changes in all the involved input parameters (reservoir characteristics and operational constraints) in few seconds. This can be accomplished for reservoir simulation models that take hours or days to make a single run. The objective of this study is to develop a new proxy model that can substitute numerical simulation and give results in a shorter period of time while maintaining accuracy.

3.2 Research Objective

The main objectives of this study are:

1. Develop a simple proxy model for reservoir simulation that can describe the performance of the wells at any point in time during the simulation period.
2. Evaluate the proposed proxy model in terms of:
    - Accuracy in the results with respect to the conventional numerical reservoir simulation.
    - Computational cost
    - Speed

Benefits of this study are as follows:

1. The proposed model will be another option for primary evaluation and estimations beside numerical reservoir simulation.
2. The proposed model will be an inexpensive alternative for numerical reservoir simulation in developmental studies and optimization processes.

3.3 Methodology

The proposed methodology is by following the Engineering control in reservoir modeling. Engineering control is defined as "Degree of Understanding of the reservoir as a physical system described by its static and dynamic parameters and the Ability to Exert Control over its future performance". The reservoir model is the translation of engineering control as a tool that can be used to manage the performance of the reservoir. The effectiveness of any model is limited by the degree of how much this model is a true representation of all the processes that happen in the reservoir. Understanding how much confidence we have in the given reservoir parameters and following a controlled methodology to model the reservoir we can achieve a higher degree of control over the reservoir performance.

To construct a reservoir model, a frame of reference using international best practice was developed, the start is by establishing key steps for reservoir modeling as follows:

- State the objectives
- Characterization
- Model selection
- Model construction
- Validation
- Application

3.3.1 Single phase

As mentioned, the objective of this study is to create a proxy model that can mimic and replace the full reservoir numerical model. The model should outperform traditional numerical simulation in speed while maintaining accuracy.

The main downfall of full reservoir numerical simulation in the aspect of speed that is solving the diffusivity equation in grid to grid base, so as the number of grids increases the time for the simulation process increases. Also the different methods of solutions for the system of equation, like iterative methods (i.e. Newton method) or Implicit Pressure Explicit Saturation IMPES, take periods of time to converge. The main idea in this work is to reduce the number of computations, which is explained later in this chapter. Another thing is to come up with a system of equations that can be solved directly without going to iterative methods. Thus reduce the time required for the simulation.

The first step is to identify all the features and parameters affecting the reservoir performance that can be used to fully describe the reservoir and construct the model. In this study the focus is on the main points affecting the potential in the reservoir, and thus affecting the fluid flow. Wells, either producers or injectors, are the main source of potential difference in the reservoir, another source is the boundaries around the reservoir. By constructing a set of equations that can relate the potential difference in these points and how they affect each other, we can describe the total potential in the reservoir.

The selection is to set the type of the model whether it is 1D, 2D or 3D model for single phase, 2-phase or 3-phase. This step determines the degree of complexity of the model, as more dimensions and more phases to handle the more complex the model will be. Another selection criteria is porosity, so the model may vary between single continuum (matrix porosity), double continuum (matrix and fractures or matrix and vugs) or triple continuum (matrix, fractures and vugs).

This study focus on a reservoir model of 3D single phase single continuum as a base case that can be adjusted and improved to describe more complex models. As the constructed model achieves its objective, then more complexity is introduced to the model in order to describe as much as possible reservoir cases.

Transformation of all the parameters in the characterization step into a simulation model depends on the scale for those parameters, and this step is the main source of error. Reservoir models are not unique; as more than one model can match the same reservoir performance. The main point is that the number of unknowns to be estimated should be commensurate with the number of the known measurements that we already have.

In this work, firstly, starting from the diffusivity equation:

$$-\nabla \bullet \left(\rho_j \vec{u}_j \right) + \frac{\rho_j q_j}{V_b} = \frac{\partial \left(\phi \rho_j S_j \right)}{\partial t} \tag{1}$$

Where j represent the flowing phase, all the other parameters as commonly defined.

The velocity vector $\vec{u}_j$ is defined as:

$$\vec{u}_j = -\frac{k_{rj}}{\mu_j} \bar{\bar{k}} \bullet \nabla \Phi_j \tag{2}$$

Where Φ_j represent the potential which is defined as:

$$\Phi_j = p_j - \rho_j g z \tag{3}$$

By substituting we obtain:

$$\nabla \bullet \left[\rho_j \frac{k_{rj}}{\mu_j} \bar{\bar{k}} \bullet \nabla \left(p_j - \rho_j g z \right) \right] + \frac{\rho_j q_j}{V_b} = \frac{\partial \left(\phi \rho_j S_j \right)}{\partial t} \tag{4}$$

Considering one dimension follow, the previous equation becomes:

$$\frac{\partial}{\partial x}\left[\rho_j \frac{k_{rj}}{\mu_j}\bar{\bar{k}}\bullet\left(\frac{\partial p_j}{\partial x}-\rho_j g\frac{\partial z}{\partial x}\right)\right]+\frac{\rho_j q_j}{V_b}=\frac{\partial\left(\phi\rho_j S_j\right)}{\partial t} \tag{5}$$

For a single phase flow and assuming there is an average permeability that can represent the flow, by dividing by ρ_{sc} we reach to:

$$\frac{\partial}{\partial x}\left[\frac{k}{\mu B}\left(\frac{\partial p}{\partial x}-\rho g\frac{\partial z}{\partial x}\right)\right]+\frac{q_{sc}}{V_b}=\frac{\partial}{\partial t}\left(\frac{\phi}{B}\right) \tag{6}$$

Having the compressibility defined for both matrix and fluid as:

$$c_r=\frac{1}{\phi}\frac{d\phi}{dp}$$

$$c_f=-\frac{1}{B}\frac{dB}{dp} \tag{7}$$

The total compressibility is:

$$c_t=c_f+c_r \tag{8}$$

By substituting in the equation (6)

$$\frac{\partial}{\partial x}\left[\frac{V_b k}{\mu B}\left(\frac{\partial p}{\partial x}-\rho g\frac{\partial z}{\partial x}\right)\right]+q_{sc}=\frac{V_b\phi c_t}{B}\frac{\partial p}{\partial t} \tag{9}$$

If we have:

$$V_b=\Delta x\Delta y\Delta z \tag{10}$$

$$A_x=\Delta y\Delta z=A \tag{11}$$

By using finite difference method (FDM), equation (9) simplifies to:

$$\frac{A_x k}{\mu B}\left(\frac{\Delta p}{\Delta x}-\rho g\frac{\Delta z}{\Delta x}\right)+q_{sc}=\frac{V_b\phi C_t}{B}\frac{\Delta p}{\Delta t} \tag{12}$$

Using an explicit method of solution, we can obtain a discrete equation as:

$$\frac{A_x k}{\mu B}\left[\frac{\left(p_{x+\Delta x}-p_x\right)}{\Delta x}-\rho g\frac{\left(z_{x+\Delta x}-z_x\right)}{\Delta x}\right]+q_{sc}=\frac{V_b \phi C_t}{B}\frac{\left(p_x^{n+1}-p_x^n\right)}{\Delta t} \tag{13}$$

$$\frac{T}{\Delta x}\left[\left(p_{x+\Delta x}-p_x\right)-\rho g\left(z_{x+\Delta x}-z_x\right)\right]+q_{sc}=\frac{C_h}{\Delta t}\left(p_x^{n+1}-p_x^n\right) \tag{14}$$

Where

$$T=\frac{A_x k}{\mu B} \tag{15}$$

And

$$C_h=\frac{V_b \phi C_t}{B} \tag{16}$$

By rearranging and generalizing for any point in any direction, making the pressure drop as the subject of the equation we have the following equation:

$$\left(p_m^{n+1}-p_m^n\right)=\frac{\Delta t}{C_h}\left\{\frac{T}{\Delta L}\left[\left(p_{m+\Delta L}-p_m\right)-\rho g\left(z_{m+\Delta L}-z_m\right)\right]+q_{sc}\right\} \tag{17}$$

This equation relates the pressure drop in one point of the reservoir to the flow rate at the same point, and the pressure at another point in the reservoir considering the fluid and rock properties in the space separating the two points.

The model proposed in this work tries to reduce the computations by constructing a network between the main points discussed earlier. Each connection contains rock and fluid properties that can describe the interaction between each pair of points, these properties include transmissibility and distance. Figure 3.1 and Figure 3.2 illustrate the construction of the model connection.

Considering the location of a well with respect to the reservoir boundaries, a connection is constructed between the objective well and each boundary. This procedure is then repeated for each well in the reservoir in order to link all the wells with the reservoir boundaries.

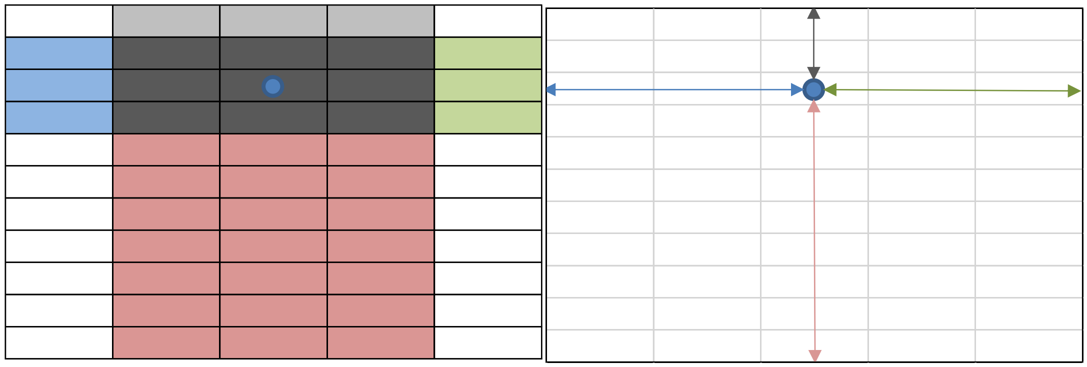

Figure 3.1: Model connections between a well and reservoir boundaries

Considering the location of a well with respect to another well, another connection is constructed between these two wells. And this procedure is repeated for the objective well with all other wells in the reservoir, then repeated for all the other wells.

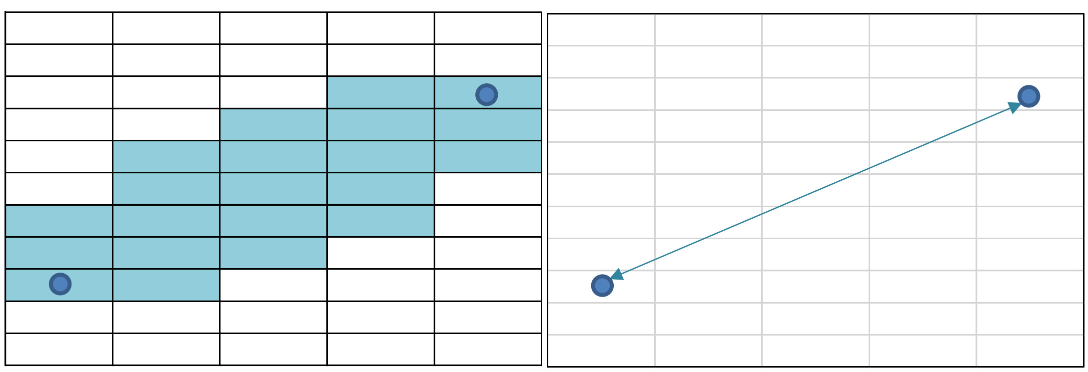

Figure 3.2: Model connection between a pair of wells

As shown in Figure 3.1 and Figure 3.2, the permeability of the grid blocks highlighted along the connection are used to calculate the transmissibility represented in Equation (15) by taking the average of their values. Following these procedures, if the reservoir contains three wells the model network of connections will be as illustrated in Figure 3.3.

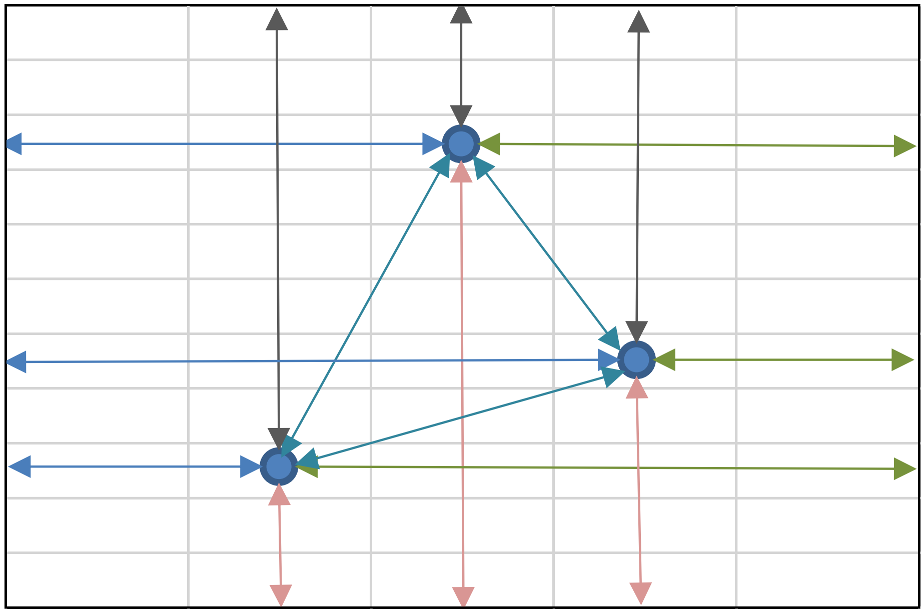

Figure 3.3: Model network of connections for the whole reservoir

By rearranging equation [13], we can have:

$$\left(p_m^{n+1}-p_m^n\right)=\frac{\Delta tB}{V_b\phi C_t}\left\{\frac{Ak}{\mu B\Delta L}\left[\left(p_{m+\Delta L}-p_m\right)-\rho g\left(z_{m+\Delta L}-z_m\right)\right]+q_{sc}\right\} \tag{18}$$

After generating all the connections between the wells, for each well we generate a control pore volume by taking half of the mean of all the distances to the other wells as the length of a square shape with the objective well in the center of it.

By looking into equation [18], considering the proportionate of all the rock and fluid properties with the pressure drop, we can describe the pressure drop in any point in the reservoir with respect to the production rate of that point, and the pressure at any other point in the reservoir at a specified distance and having a single value that can represent the transmissibility between them. Some parameters were introduced to this equation in account for the averaging process and assumptions considered. These parameters will be calculated using DE with specified bounds to find the best values. Equation (18) can become:

$$\left(p_m^{n+1}-p_m^n\right)=\frac{a_1\Delta t^{a_2}B}{V_b\phi C_t}\left\{\frac{Ak^{a_3}}{\mu B\Delta L^{a_4}}\left[\left(p_{m+\Delta L}-p_m\right)-\rho g\left(z_{m+\Delta L}-z_m\right)\right]+q_{sc}{}^{a_5}\right\} \tag{19}$$

By taking account for the effects of all other points in the reservoir on an objective point, equation [19] will become:

$$\left(p_m^{n+1}-p_m^n\right)=\frac{a_1\Delta t^{a_2}B}{V_b\phi C_t}\left\{\sum_{i=1}^{N.p}\frac{Ak^{a_3}}{\mu B\Delta L^{a_4}}\left[\left(\left(p_{m+\Delta L}\right)_i-p_m\right)-\rho g\left(\left(z_{m+\Delta L}\right)_i-z_m\right)\right]+q_{sc}{}^{a_5}\right\} \tag{20}$$

Where (N.p) is the number of the neighboring points. In equation (20) the term $\left(\left(p_{m+\Delta L}\right)_i-p_m\right)$ can be used to account for the effect of the neighboring wells on the objective well, as well as it can be used to account for the effect of the boundary condition. Regarding the type of the boundary condition, if we have constant head boundary condition then $p_{x+\Delta x}=p_{boundary}$. If we have a constant flux boundary condition, from fluid flow concepts we know that:

$$q_{sc}=\frac{Ak}{\mu B}\left(\frac{\Delta p}{\Delta L}\right)=\frac{Ak}{\mu B\Delta L}\left(p_{m+\Delta L}-p_m\right) \tag{21}$$

So it is possible to substitute in equation [20] for the effect of the boundary on the objective point as:

$$\frac{Ak^{a_3}}{\mu B\Delta L^{a_4}}\left(p_{m+\Delta L}-p_m\right)=q_{boundary} \tag{22}$$

Equation [20] represents the proposed model of this study, and to simulate the whole reservoir performance this equation should be solved for every point (well) with respect to all other points (wells and boundaries) in the reservoir. This will lead to a set of equations that need to be solved at every time step. In this study the set of equations will be solved explicitly as:

$$\left(p_m^{n+1}-p_m^n\right)=\frac{a_1\Delta t^{a_2}B}{V_b\phi C_t}\left\{\sum_{i=1}^{N.p}\frac{Ak^{a_3}}{\mu B\Delta L^{a_4}}\left[\left(\left(p_{m+\Delta L}^n\right)_i-p_m^n\right)-\rho g\left(\left(z_{m+\Delta L}\right)_i-z_m\right)\right]+q_{sc}{}^{a_5}\right\} \tag{23}$$

And implicitly as:

$$\left(p_m^{n+1} - p_m^n\right) = \frac{a_1 \Delta t^{a_2} B}{V_b \phi C_t} \left\{ \sum_{i=1}^{N.p} \frac{A k^{a_3}}{\mu B \Delta L^{a_4}} \left[\left(\left(p_{m+\Delta L}^{n+1} \right)_i - p_m^{n+1} \right) - \rho g \left(\left(z_{m+\Delta L} \right)_i - z_m \right) \right] + q_{sc}{}^{a_5} \right\} \quad (24)$$

A comparison between the two approaches will be conducted to select the one that gives a better description for the reservoir performance.

As for the explicit approach, it will not see the change in the flow rate of a point until the next step; so an adjustment is proposed to encounter for this case by introducing a new term ($\frac{q_{sc}^{n+1}}{q_{sc}^n}$) to the model relating the previous and the current rates of a specific point, so the final explicit model can be described by:

$$\left(p_m^{n+1} - p_m^n\right) = \frac{a_1 \Delta t^{a_2} B}{V_b \phi C_t} \left\{ \sum_{i=1}^{N.p} \frac{A k^{a_3}}{\mu B \Delta L^{a_4}} \left[\left(\frac{q_{sc}^{n+1}}{q_{sc}^n} \right)^{a_6} \left(\left(p_{m+\Delta L}^{n} \right)_i - p_m^{n} \right) - \rho g \left(\left(z_{m+\Delta L} \right)_i - z_m \right) \right] + q_{sc}{}^{a_5} \right\} \quad (25)$$

Starting from the initial reservoir and operational conditions, first part of the history data or simulation data is used to train the proposed model in order to determine the model parameters. Differential Evolution DE technique is used to optimize the model, and find the best values for the model parameters that can give a better match to the training data set. During this optimization process the model results are constrained by some objective function to minimize the error with respect to the training data. After all the model coefficients and parameters are set and quantified, then the model results are matched with the rest of the available history or simulation data to conclude the validation step.

Not like artificial intelligent proxy models or other techniques that need to be trained many times as the conditions change, the proposed model in this study need to be trained only one time and then it can adjust itself to any change in the reservoir conditions or operational conditions.

For operational conditions, in the case of rate constrain, given the value for the bottom hole flowing pressure, the model checks for the value of the flow rate if it violates the constrain or not. If the calculated rate does not violate the rate constrain the model calculate the pressure at the well, else the model will adjust the value of the rate to the constrain value and use it to calculate the pressure in the well. In the case of pressure constrain, given the value for the flow rate, the model checks for the bottom hole flowing pressure. If the bottom hole pressure violates the pressure constrain the model will adjust the value of the bottom hole pressure to the constrain value and calculate the flow rate according to this value, if it does not violate the pressure constrain the model will proceed to calculate the pressure at the specific well.

3.3.2 Two phase

Starting from equation (4):

$$\frac{\partial}{\partial x}\left[\rho_j \frac{k_{rj}}{\mu_j}\bar{\bar{k}}\bullet\left(\frac{\partial p}{\partial x}-\rho_j g\frac{\partial z}{\partial x}\right)\right]+\frac{\rho_j q_j}{V_b}=\frac{\partial\left(\phi\rho_j S_j\right)}{\partial t} \tag{4}$$

Diving the whole equation by the density of the denoted phase we get to:

$$\frac{\partial}{\partial x}\left[\frac{k_{rj}}{\mu_j B_j}\bar{\bar{k}}\bullet\left(\frac{\partial p}{\partial x}-\rho_j g\frac{\partial z}{\partial x}\right)\right]+\frac{q_{j_{sc}}}{V_b}=\frac{\partial}{\partial t}\left(\frac{\phi S_j}{B_j}\right) \tag{26}$$

By rearranging:

$$\frac{\partial}{\partial x}\left[\frac{V_b k_{rj}}{\mu_j B_j}\bar{\bar{k}}\bullet\left(\frac{\partial p}{\partial x}-\rho_j g\frac{\partial z}{\partial x}\right)\right]+q_{j_{sc}}=V_b\frac{\partial}{\partial t}\left(\frac{\phi S_j}{B_j}\right) \tag{27}$$

If we have:

$$V_b=\Delta x\Delta y\Delta z \tag{28.a}$$

$$A_x=\Delta y\Delta z=A \tag{28.b}$$

By substituting (28.a) and (28.b) in (27) and using finite difference we can get to:

$$\frac{A_x k_{rj}}{\mu_j B_j}\bar{\bar{k}}\bullet\left(\frac{\Delta p}{\Delta x}-\rho_j g\frac{\Delta z}{\Delta x}\right)+q_{j_{sc}}=V_b\frac{\Delta}{\Delta t}\left(\frac{\phi S_j}{B_j}\right) \tag{29}$$

This equation can be written for both the two phases exist in the reservoir (oil, water), by denoting o for oil and w for water as follows:

$$\frac{A_x k_{ro}}{\mu_o B_o}\bar{\bar{k}}\bullet\left(\frac{\Delta p_o}{\Delta x}-\rho_o g\frac{\Delta z}{\Delta x}\right)+q_{o_{sc}}=V_b\frac{\Delta}{\Delta t}\left(\frac{\phi S_o}{B_o}\right) \tag{30}$$

$$\frac{A_x k_{rw}}{\mu_w B_w}\bar{\bar{k}}\bullet\left(\frac{\Delta p_w}{\Delta x}-\rho_w g\frac{\Delta z}{\Delta x}\right)+q_{w_{sc}}=V_b\frac{\Delta}{\Delta t}\left(\frac{\phi S_w}{B_w}\right) \tag{31}$$

$$\frac{T_o}{\Delta x}\left[\left(p_{o_{x+\Delta x}}-p_{o_x}\right)-\rho_o g\left(z_{x+\Delta x}-z_x\right)\right]+q_{o_{sc}}=\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_o}{B_o}\right) \tag{32}$$

$$\frac{T_w}{\Delta x}\left[\left(p_{w_{x+\Delta x}}-p_{w_x}\right)-\rho_w g\left(z_{x+\Delta x}-z_x\right)\right]+q_{w_{sc}}=\frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_w}{B_w}\right) \tag{33}$$

Where

$$T_o=\frac{A_x k_{ro} k}{\mu_o B_o} \tag{34.a}$$

$$T_w=\frac{A_x k_{rw} k}{\mu_w B_w} \tag{34.b}$$

The main focus in solving these equation is to make it as functions of our objective parameters, oil pressure and water saturation. In order to achieve that the following relations are considered. First the relation between the oil pressure and the water pressure in terms of capillary pressure presented as:

$$p_c = p_o - p_w \tag{35.a}$$

Then it is possible to write:

$$p_w = p_o - p_c \tag{35.b}$$

For the oil saturation and the water saturation, it is only two phase flow so it is possible to write:

$$S_o + S_w = 1 \tag{36.a}$$

Then it can be written as:

$$S_o = 1 - S_w \tag{36.b}$$

By substituting in the equations and make them general for any point in any direction:

$$\frac{T_o}{\Delta L}\left[\left(p_{o_{m+\Delta L}} - p_{o_x}\right) - \rho_o g\left(z_{m+\Delta L} - z_m\right)\right] + q_{o_{sc}} = \frac{V_b}{\Delta t}\Delta_t\left[\frac{\phi(1-S_w)}{B_o}\right] \tag{37}$$

$$\frac{T_w}{\Delta L}\left[\left(p_{o_{m+\Delta L}} - p_{o_m}\right) - \left(p_{c_{m+\Delta L}} - p_{c_m}\right) - \rho_w g\left(z_{m+\Delta L} - z_m\right)\right] + q_{w_{sc}} = \frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_w}{B_w}\right) \tag{38}$$

Taking in consideration all the points in the reservoir discussed earlier, for each point it is possible to write:

$$\sum_{i=1}^{N.p}\left\{\frac{T_o}{\Delta L}\left[\left(p_{o_{m+\Delta L}} - p_{o_m}\right) - \rho_o g\left(z_{m+\Delta L} - z_m\right)\right]\right\} + q_{o_{sc}} = \frac{V_b}{\Delta t}\Delta_t\left[\frac{\phi(1-S_w)}{B_o}\right] \tag{39}$$

$$\sum_{i=1}^{N.p}\left\{\frac{T_w}{\Delta L}\left[\left(p_{o_{m+\Delta L}} - p_{o_m}\right) - \left(p_{c_{m+\Delta L}} - p_{c_m}\right) - \rho_w g\left(z_{m+\Delta L} - z_m\right)\right]\right\} + q_{w_{sc}} = \frac{V_b}{\Delta t}\Delta_t\left(\frac{\phi S_w}{B_w}\right) \tag{40}$$

As mentioned before the solution of all equations in these work is in implicit form. So, in order to solve these equations, it is proposed to use Newton Raphson iterative method. At each point there are two unknowns, pressure and water saturation, which are written at every time step in the calculation as:

$$p_m^{n+1} = p_m^n + \delta p_m^{n+1} \tag{41}$$

$$S_m^{n+1} = S_m^n + \delta S_m^{n+1} \tag{42}$$

Then the formulation of the residual equations for the Newton Raphson method can be written based on the equations that we have as follows:

Oil Residual

$$R_{o_m} = \sum_{i=1}^{N.p} \left\{ \frac{T_o}{\Delta L} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - \rho_o g \left(z_{m+\Delta L} - z_m \right) \right] \right\} + q_{o_{sc}} - \frac{V_b}{\Delta t} \Delta_t \left[\frac{\phi(1-S_w)}{B_o} \right] \tag{43}$$

Water Residual

$$R_{w_m} = \sum_{i=1}^{N.p} \left\{ \frac{T_w}{\Delta L} \left[\left(p_{o_{m+\Delta L}} - p_{o_L} \right) - \left(p_{c_{m+\Delta L}} - p_{c_m} \right) - \rho_w g \left(z_{m+\Delta L} - z_m \right) \right] \right\} + q_{w_{sc}} - \frac{V_b}{\Delta t} \Delta_t \left(\frac{\phi S_w}{B_w} \right) \tag{44}$$

Now it is possible to define the unknown vector and the residual vector as:

$$y = \begin{pmatrix} p_o \\ S_w \end{pmatrix} \tag{45}$$

$$R = \begin{pmatrix} R_o \\ R_w \end{pmatrix} \tag{46}$$

Then the application of Newton Raphson iterative method of solution yields a set of equations in terms of δp and δS, which can be written generally in terms of ∂y.

$$\frac{\partial R}{\partial y} = \begin{pmatrix} \frac{\partial R_w}{\partial p_o} & \frac{\partial R_w}{\partial S_w} \\ \frac{\partial R_o}{\partial p_o} & \frac{\partial R_o}{\partial S_w} \end{pmatrix} \tag{47}$$

Taking partial derivatives of the residuals with respect to the pressure at the objective point and the other neighboring points, we get:

$$\frac{\partial R_{o_m}}{\partial p_{o_m}} = \sum_{i=1}^{N.p} \left\{ \begin{array}{l} \frac{1}{\Delta L} \frac{\partial T_o}{\partial p_{o_m}} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - \gamma_o \left(z_{m+\Delta L} - z_m \right) \right] \\ + \frac{T_o}{\Delta L} \left[-1 - \frac{\partial \gamma_o}{\partial p_{o_m}} \left(z_{m+\Delta L} - z_m \right) \right] \end{array} \right\} + \frac{\partial q_{o_{sc}}}{\partial p_{o_m}} - \frac{V_b}{\Delta t} \Delta_t \frac{\partial \left[\frac{\phi(1-S_w)}{B_o} \right]}{\partial p_{o_m}} \tag{48}$$

$$\frac{\partial R_{w_m}}{\partial p_{o_m}} = \sum_{i=1}^{N.p} \left\{ \begin{array}{l} \dfrac{1}{\Delta L} \dfrac{\partial T_w}{\partial p_{o_m}} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - p'_{c_{m,m+\Delta L}} \left(S_{m+\Delta L} - S_m \right) - \gamma_w \left(z_{m+\Delta L} - z_m \right) \right] \\ + \dfrac{T_w}{\Delta L} \left[-1 - \dfrac{\partial \gamma_w}{\partial p_{o_m}} \left(z_{m+\Delta L} - z_m \right) \right] \end{array} \right\} \tag{49}$$

$$+ \frac{\partial q_{w_{sc}}}{\partial p_{o_m}} - \frac{V_b}{\Delta t} \Delta_t \frac{\partial \left(\dfrac{\phi S_w}{B_w} \right)}{\partial p_{o_m}}$$

$$\frac{\partial R_{o_m}}{\partial p_{o_{m+\Delta L}}} = \sum_{i=1}^{N.p} \left\{ \begin{array}{l} \dfrac{1}{\Delta L} \dfrac{\partial T_o}{\partial p_{o_{m+\Delta L}}} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - \gamma_o \left(z_{m+\Delta L} - z_m \right) \right] \\ + \dfrac{T_o}{\Delta L} \left[1 - \dfrac{\partial \gamma_o}{\partial p_{o_{m+\Delta L}}} \left(z_{m+\Delta L} - z_m \right) \right] \end{array} \right\} \tag{50}$$

$$\frac{\partial R_{w_m}}{\partial p_{o_{m+\Delta L}}} = \sum_{i=1}^{N.p} \left\{ \begin{array}{l} \dfrac{1}{\Delta L} \dfrac{\partial T_w}{\partial p_{o_{m+\Delta L}}} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - p'_{c_{m,m+\Delta L}} \left(S_{m+\Delta L} - S_m \right) - \gamma_w \left(z_{m+\Delta L} - z_m \right) \right] \\ + \dfrac{T_w}{\Delta L} \left[1 - \dfrac{\partial \gamma_w}{\partial p_{o_{m+\Delta L}}} \left(z_{m+\Delta L} - z_m \right) \right] \end{array} \right\} \tag{51}$$

Now taking partial derivatives of the residuals with respect to the water saturation at the objective point and the other neighboring points, we get:

$$\frac{\partial R_{o_m}}{\partial S_{w_m}} = \sum_{i=1}^{N.p} \left\{ \frac{1}{\Delta L} \frac{\partial T_o}{\partial S_{w_m}} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - \gamma_o \left(z_{m+\Delta L} - z_m \right) \right] \right\} + \frac{\partial q_{o_{sc}}}{\partial S_{w_m}} - \frac{V_b}{\Delta t} \Delta_t \frac{\partial \left[\dfrac{\phi \left(1 - S_w \right)}{B_o} \right]}{\partial S_{w_m}} \tag{52}$$

$$\frac{\partial R_{w_m}}{\partial S_{w_m}} = \sum_{i=1}^{N.p} \left\{ \begin{array}{l} \dfrac{1}{\Delta L} \dfrac{\partial T_w}{\partial S_{w_m}} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - p'_{c_{m,m+\Delta L}} \left(S_{m+\Delta L} - S_m \right) - \gamma_w \left(z_{m+\Delta L} - z_m \right) \right] \\ + \dfrac{T_w}{\Delta L} \left[- p''_{c_{m,m+\Delta L}} \left(S_{m+\Delta L} - S_m \right) + p'_{c_{m,m+\Delta L}} - \dfrac{\partial \gamma_w}{\partial S_{w_m}} \left(z_{m+\Delta L} - z_m \right) \right] \end{array} \right\} \tag{53}$$

$$+ \frac{\partial q_{w_{sc}}}{\partial S_{w_m}} - \frac{V_b}{\Delta t} \Delta_t \frac{\partial \left(\dfrac{\phi S_w}{B_w} \right)}{\partial S_{w_m}}$$

$$\frac{\partial R_{o_m}}{\partial S_{w_{m+\Delta L}}} = \sum_{i=1}^{N.p} \left\{ \frac{1}{\Delta L} \frac{\partial T_o}{\partial S_{w_{m+\Delta L}}} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - \gamma_o \left(z_{m+\Delta L} - z_m \right) \right] \right\} \tag{54}$$

$$\frac{\partial R_{w_m}}{\partial S_{w_{m+\Delta L}}} = \sum_{i=1}^{N.p} \left\{ \begin{aligned} &\frac{1}{\Delta L} \frac{\partial T_w}{\partial S_{w_{m+\Delta L}}} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - p'_{c_{m,m+\Delta L}} \left(S_{m+\Delta L} - S_m \right) - \gamma_w \left(z_{m+\Delta L} - z_m \right) \right] \\ &+ \frac{T_w}{\Delta L} \left[-p''_{c_{m,m+\Delta L}} \left(S_{m+\Delta L} - S_x \right) - p'_{c_{m,m+\Delta L}} - \frac{\partial \gamma_w}{\partial S_{w_{m+\Delta L}}} \left(z_{m+\Delta L} - z_m \right) \right] \end{aligned} \right\} \tag{55}$$

As in the single phase model, starting from the initial reservoir and operational conditions, first part of the history data or simulation data is used to train the proposed model. The same procedure of introducing model parameters to the main equation is repeated, then the following set of equations is obtained.

$$R_{o_m} = \sum_{i=1}^{N.p} \left\{ \frac{T_o^{\,a_1}}{\Delta L^{a_2}} \left[\left(p_{o_{m+\Delta L}} - p_{o_m} \right) - \rho_o g \left(z_{m+\Delta L} - z_m \right) \right] \right\} + q_{o_{sc}}^{a_4} - \frac{a_5 V_b}{\Delta t^{a_3}} \Delta_t \left[\frac{\phi \left(1 - S_w \right)}{B_o} \right] \tag{56}$$

$$R_{w_m} = \sum_{i=1}^{N.p} \left\{ \frac{T_w^{\,a_6}}{\Delta L^{a_7}} \left[\left(p_{o_{m+\Delta L}} - p_{o_L} \right) - \left(p_{c_{m+\Delta L}} - p_{c_m} \right) - \rho_w g \left(z_{m+\Delta L} - z_m \right) \right] \right\} + q_{w_{sc}}^{a_9} - \frac{a_{10} V_b}{\Delta t^{a_8}} \Delta_t \left(\frac{\phi S_w}{B_w} \right) \tag{57}$$

In this case more model parameters were used, as there is an equation for each phase. These parameters are constant, so they will not be affected by differentiating the equations with respect to the pressure and the saturation. Differential Evolution DE technique is used to find the best values for the parameters that can give a better match to the training data given. After all the model coefficients and parameters are set, the model results are matched with the rest of the available history or simulation data to validate the model.

3.3.3 Relative Permeability Model

In this study the relative permeability model used is Corey model for two phase flow Oil-Water, which can be evaluated using the following equations:

$$k_{ro} = K_{ro(S_{w\min})} \left[\frac{S_{w\max} - S_w - S_{orw}}{S_{w\max} - S_{wi} - S_{orw}} \right]^{C_o} \tag{58}$$

$$k_{rw} = K_{rw(S_{orw})}\left[\frac{S_w - S_{wcr}}{S_{w\max} - S_{wi} - S_{orw}}\right]^{C_w} \qquad (59)$$

Figure 3.4 illustrates the model and the model parameter incorporated, and at which point every parameter is evaluated.

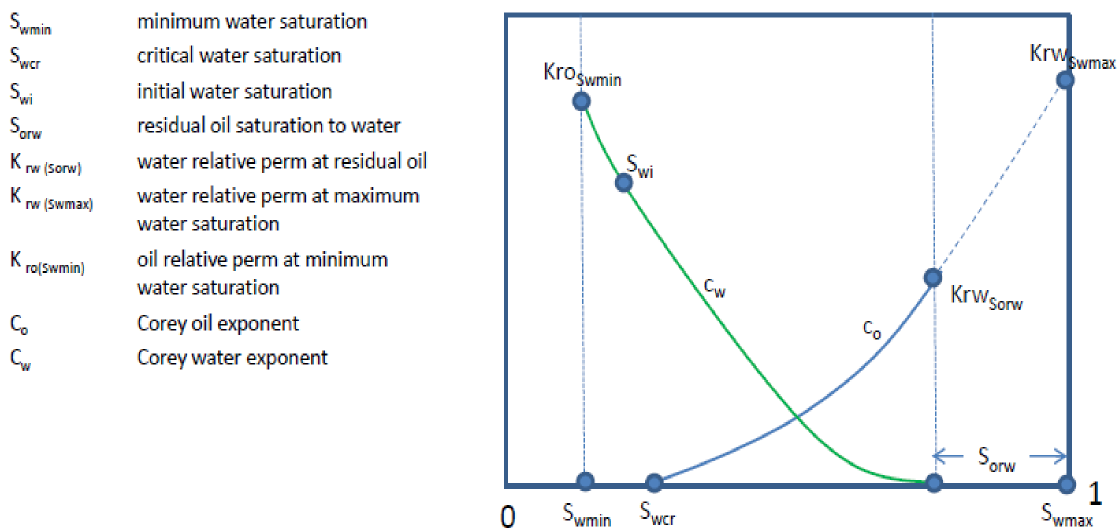

Figure 3.4:Corey model for oil - water system (Anon 2010)

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Single Phase

As there is only one phase present in this case, all the interest was focused on the value of the potential at the location of each well. In each case the error is recorded for each time step, and the maximum value of the error in each well for the entire simulation time is considered as the error for that specific well.

$$Error = \frac{\text{Simulation Results} - \text{Proxy Results}}{\text{Simulation Results}} \times 100\%$$

4.1.1 1D Flow

This example illustrates the results of the proposed proxy model versus the numerical reservoir simulation results. The case study is a single phase flow in one dimension, and the reservoir is divided into four grid blocks in the numerical simulation based on different values of the permeability, the permeability values are 50, 120, 30 and 160 respectively. The boundaries of the reservoir are all sealed (no flow boundary), except for the northern flank where there is a strong aquifer support (constant head boundary). There are two wells present in this reservoir, a producer at the first block and an injector at the third block. Figure 4.1 illustrates the reservoir in this case, and the reservoir and fluid properties are listed in Table 4.1.

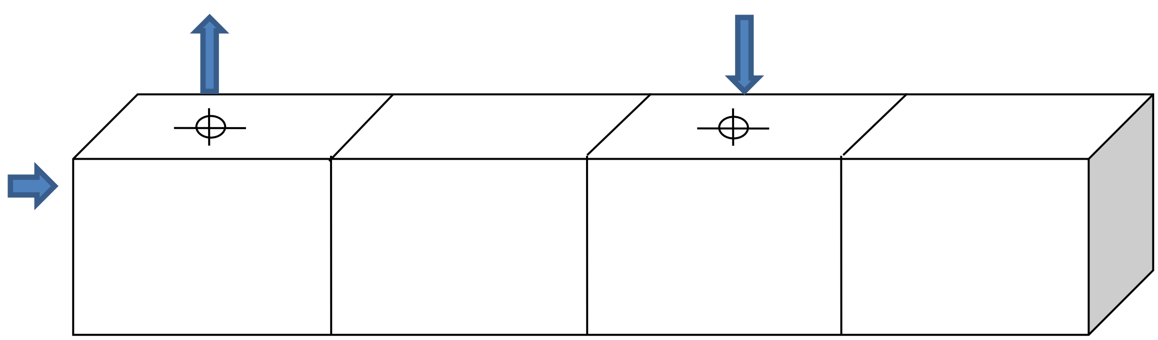

Figure 4.1: 1D Flow case

The reservoir case was simulated using numerical reservoir simulation for the duration of 100 days. Only 10% of these results were used in the training for the proxy model in order to compute the model parameters, the training results are listed in Table 4.2. The results obtained were very close to those of the numerical simulation. Table 4.3 represent the maximum relative percentage error calculated at each time step in the two wells. At the producer well there was a difference between the two methods in the transition period, but the values of the pressure were very close at the stabilization period with almost a difference of 5 psi as shown in Figure 4.2.

Table 4.1: Single phase 1D Flow reservoir and fluid properties

Reservoir dimensions	**1600*100*80**
porosity	**0.2**
Viscosity	**0.85 cp**
Formation volume factor	**1.2**
Fluid compressibility	**2E-6 psi-1**
Initial reservoir pressure	**3200 psig**
Constant head boundary pressure	**3200 psig**
Wellbore radius	**0.33 ft**
No. Wells	**2**

Table 4.2: Model parameters for Single phase 1D Flow

a1	a2	a3	a4	a5
1.04	0.93	1.31	0.98	0.87

At the injector well the results from the two methods are almost identical, and the plots of the pressure versus time were overlaying each other as displayed in Figure 4.3.

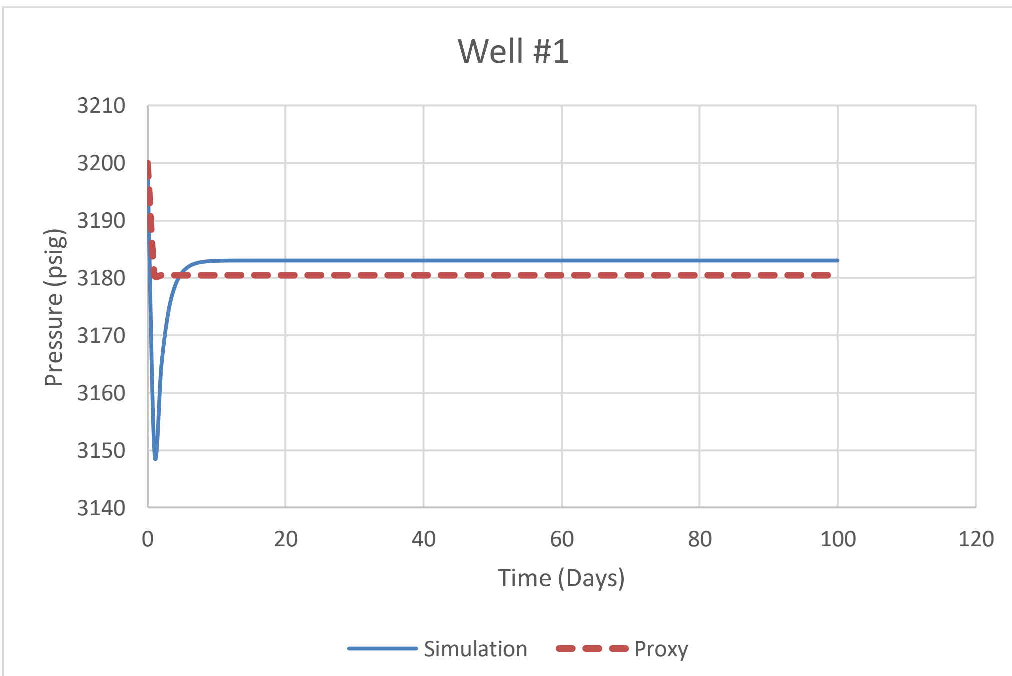

Figure 4.2: Pressure at well #1 for 1D Flow

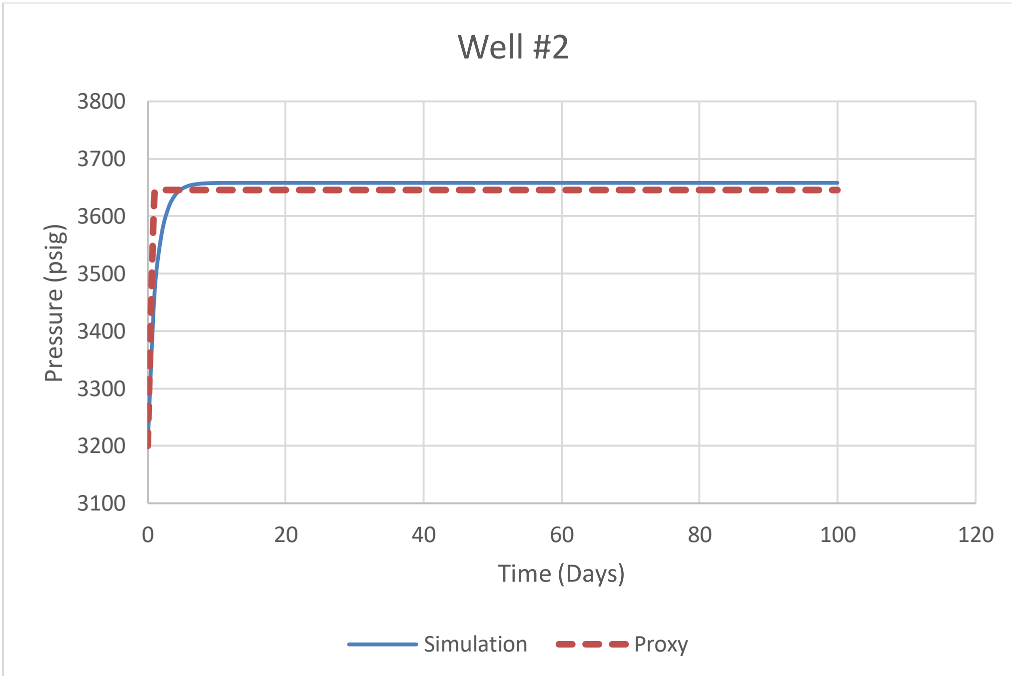

Figure 4.3: Pressure at well #2 for 1D Flow

Table 4.3: Maximum pressure error in each well for 1D Flow case

	Well #1	Well #2
Max Error%	0.98	5.06

4.1.2 2D Flow

In this example illustrate the results of the proposed proxy model are presented versus the numerical reservoir simulation results. The case study is a single phase flow in two dimensions, and the reservoir is divided into nine grid blocks in the numerical simulation based on different values of the permeability, where the permeability values range from 50 md to 240 md. The boundaries of the reservoir are no flow boundary on the eastern and western flanks, and for the northern and southern flanks there is a constant head boundary. There is a one well present in the center of the reservoir and it is illustrated in Figure 4.4.

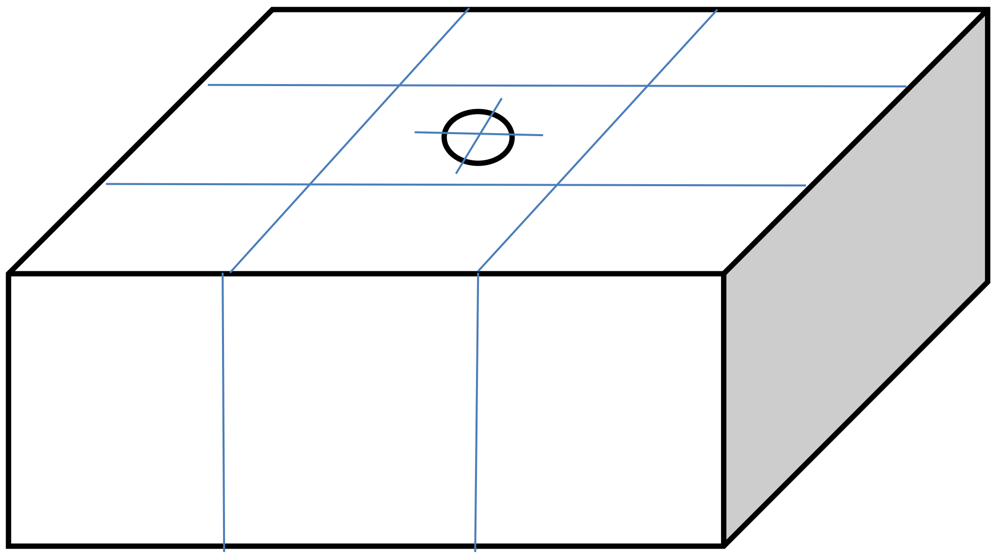

Figure 4.4: 2D Flow case

Table 4.4: Single phase 2D Flow reservoir and fluid properties

Reservoir dimensions	**1500*1800*80**
porosity	**0.2**
Viscosity	**0.85 cp**
Formation volume factor	**1.2**
Fluid compressibility	**2E-6 psi-1**
Initial reservoir pressure	**2000 psig**
Constant head boundary pressure	**4000 psig**
Wellbore radius	**0.33 ft**
No. Wells	**1**

Table 4.5: Model parameters for Single phase 2D Flow

a1	a2	a3	a4	a5
0.97	1.14	0.97	1.03	0.91

The reservoir case was simulated using numerical reservoir simulation for the duration of 100 days. Only 10% of these results were used in the training for the proxy model in order to compute the model parameters. The results obtained by the proxy model were almost identical to those of the numerical simulation, and the plots of the pressure versus time were overlaying on each other as displayed in Figure 4.5: Pressure at well #1 for 2D Flow.

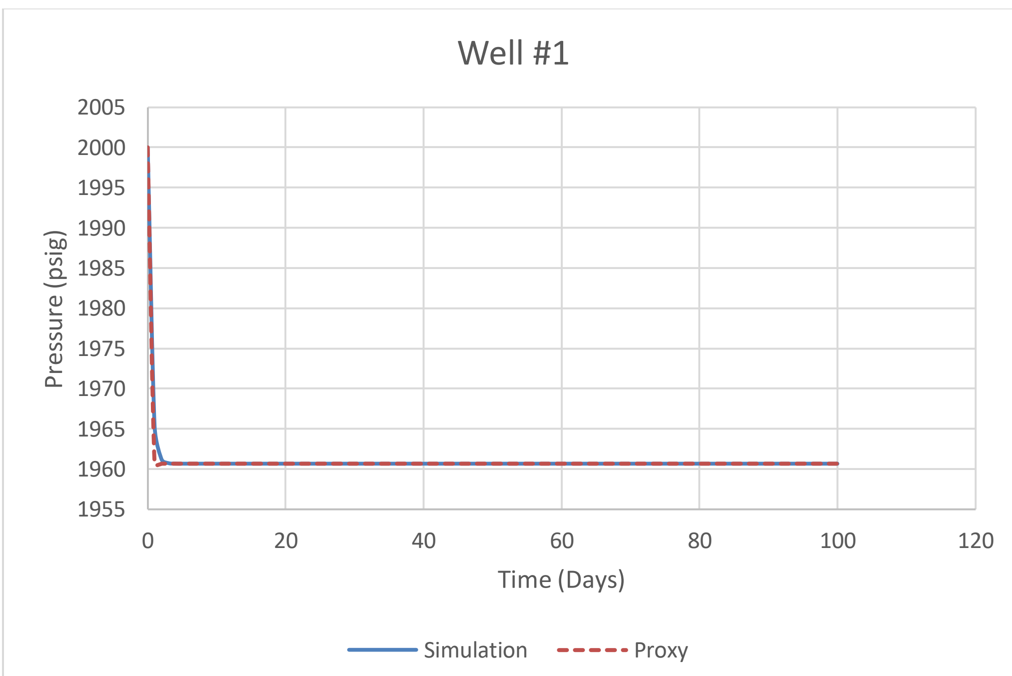

Figure 4.5: Pressure at well #1 for 2D Flow

Table 4.6: Maximum error in the well for 2D Flow case

	Well #1
Max Error%	0.24

4.1.3 3D Flow

4.1.3.1 Single Rate

This example shows the proxy model results against the traditional numerical reservoir simulation results. The case study is a single phase flow in three dimensions, and the reservoir is divided into 16x16x3 grid blocks in the numerical simulation based on different values of the permeability, where the permeability ranges between 17.41 as minimum and 3.0106e+03 for maximum. The reservoir boundary at the northern flank is sealed. There is a leak of 0.075stb/day/ft^2 into an adjacent lease along the right-side boundary line and a supply of 0.005stb/day/ft^2 to our reservoir along the southern boundary. The left-side boundary is held at a constant pressure of 4000psig. There are six wells located in the reservoir, four of them are producers and the other two are injectors as shown in Figure 4.6: 3D Flow single rate case. In this case all different types of boundary conditions are present in order to show the applicability of the proposed model for all types of boundaries. We know that the average permeability in the adjacent leases is 100md.

The results from the proxy model in all the wells were almost identical to those obtained with the numerical reservoir simulation that in some of the wells the two were overlaying on each other as shown in the figures from Figure 4.7 to Figure 4.12.

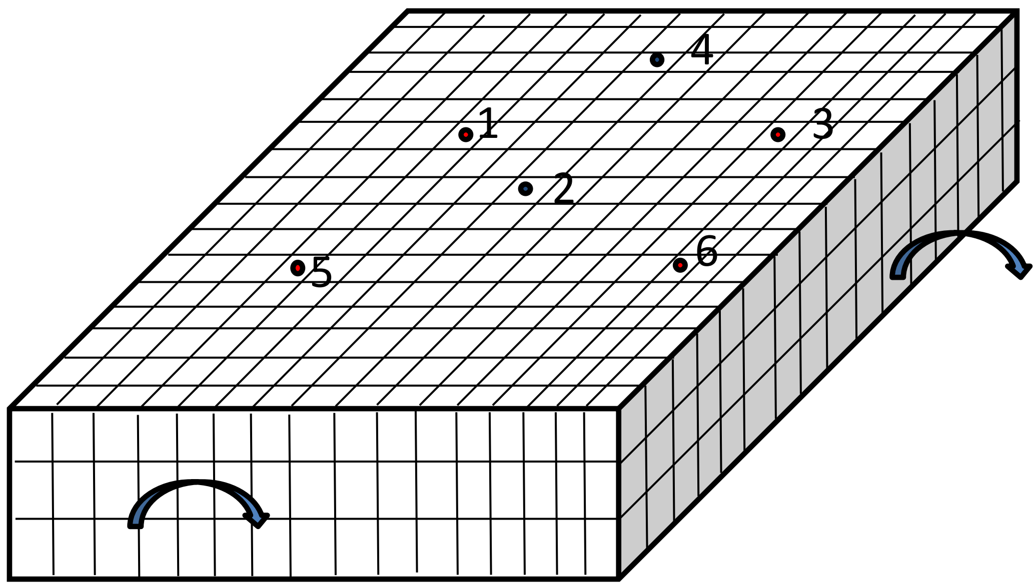

Figure 4.6: 3D Flow single rate case

Table 4.7: Single phase 3D Flow reservoir and fluid properties

Reservoir dimensions	**6400*4800*80**
porosity	**0.2**
Viscosity	**0.85 cp**
Formation volume factor	**1.2**
Fluid compressibility	**2E-6 psi-1**
Initial reservoir pressure	**4800 psig**
Constant head boundary pressure	**4000 psig**
Wellbore radius	**0.33 ft**
No. Wells	**6**

Table 4.8: Model parameters for Single phase 3D Flow

a1	a2	a3	a4	a5
9	3.98	2.973	6.233	0.535

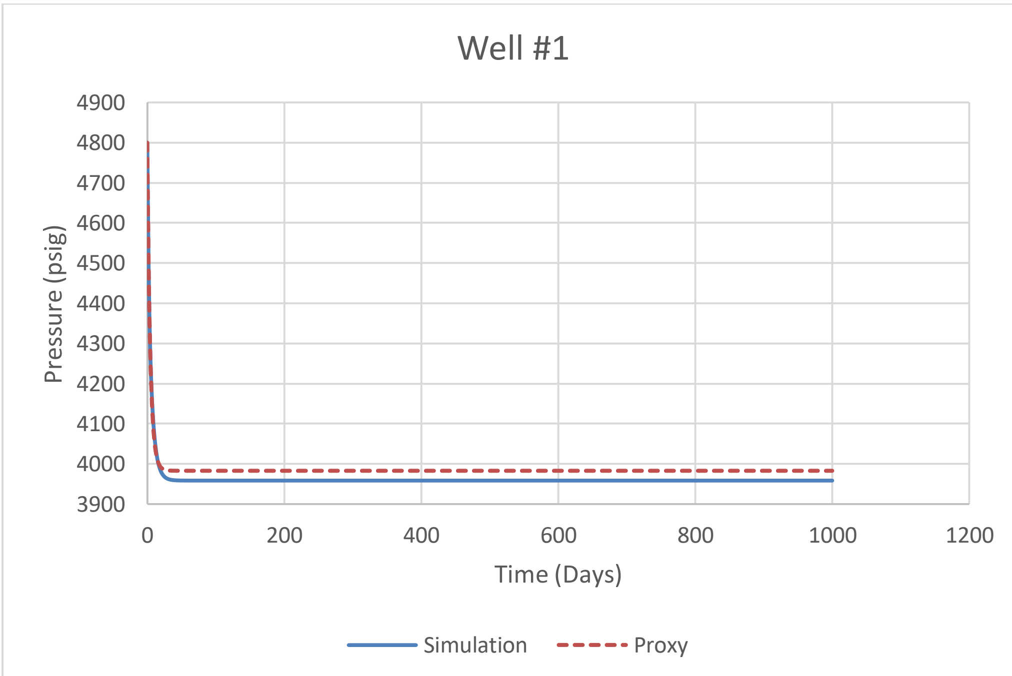

Figure 4.7: Pressure at well #1 for 3D Flow single rate

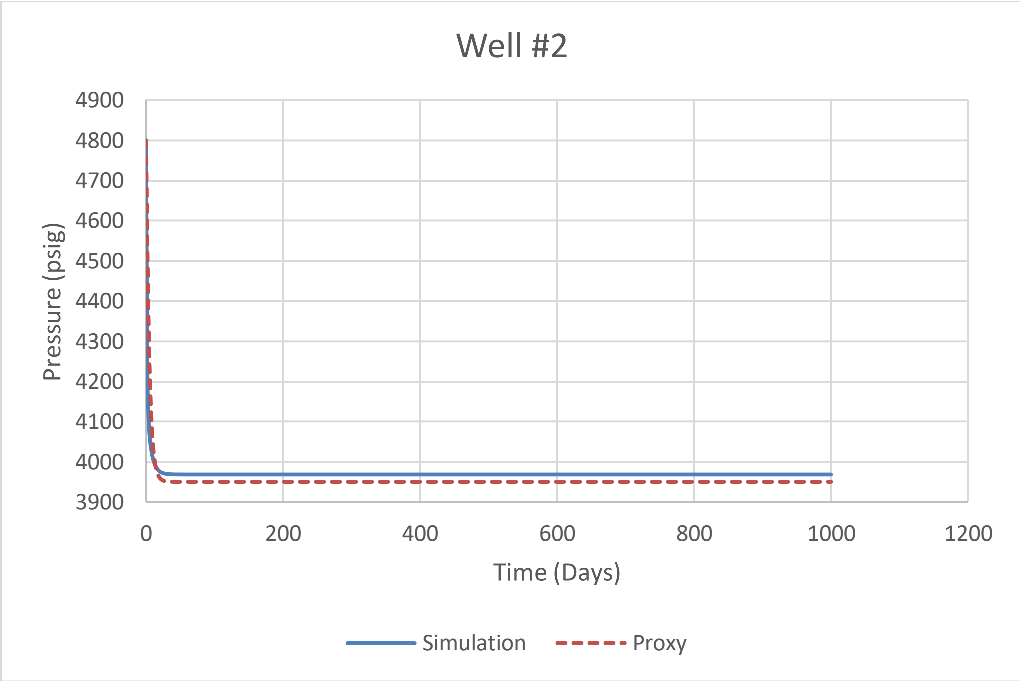

Figure 4.8: Pressure at well #2 for 3D Flow single rate

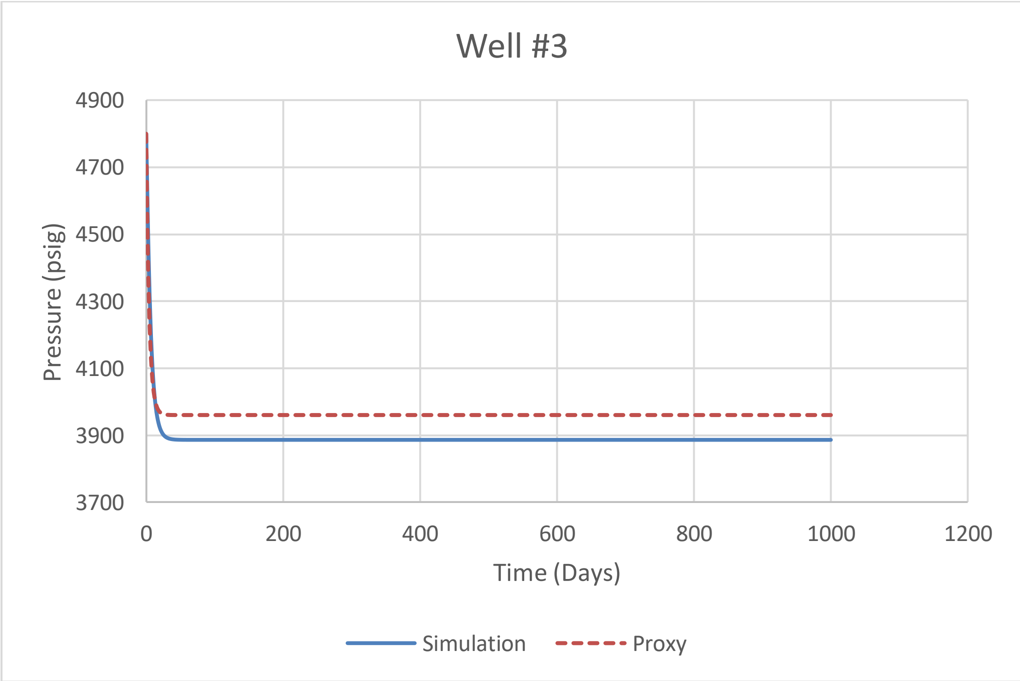

Figure 4.9: Pressure at well #3 for 3D Flow single rate

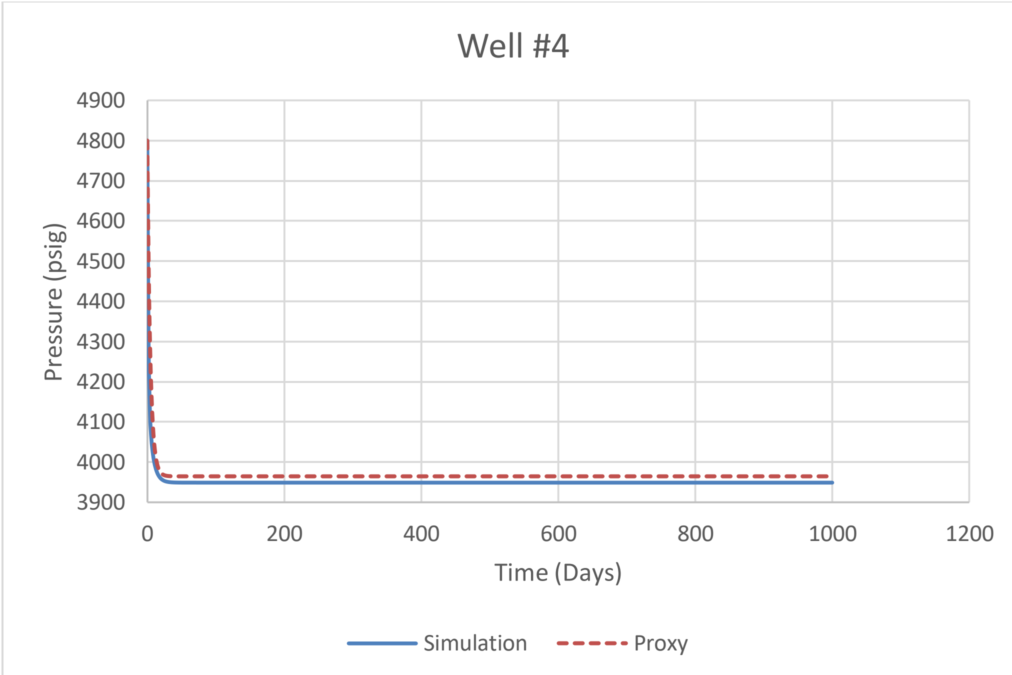

Figure 4.10: Pressure at well #4 for 3D Flow single rate

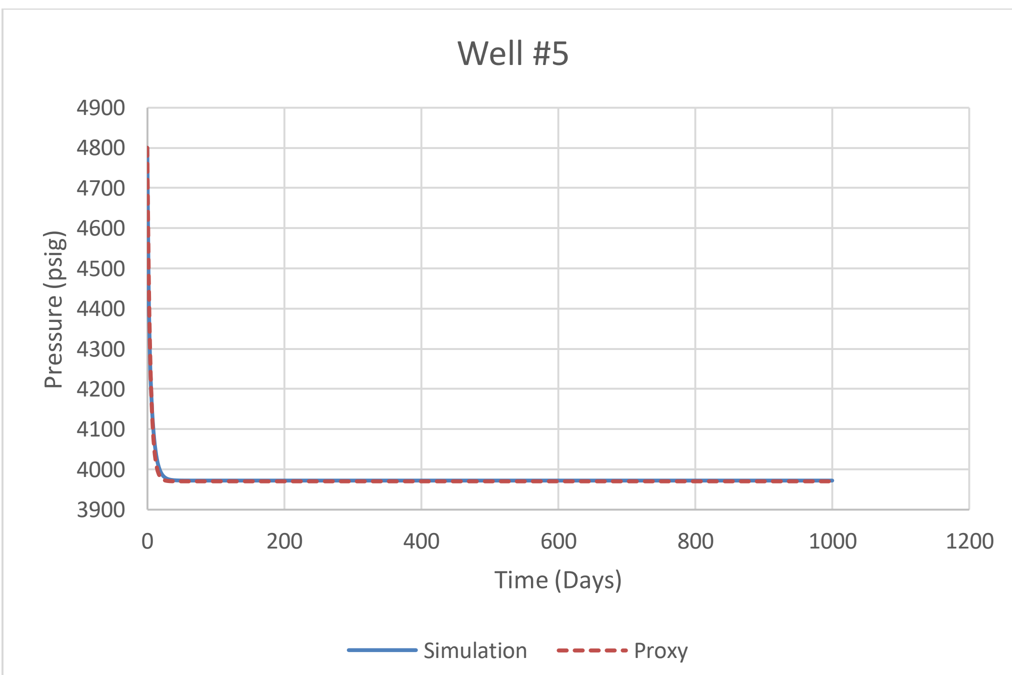

Figure 4.11: Pressure at well #5 for 3D Flow single rate

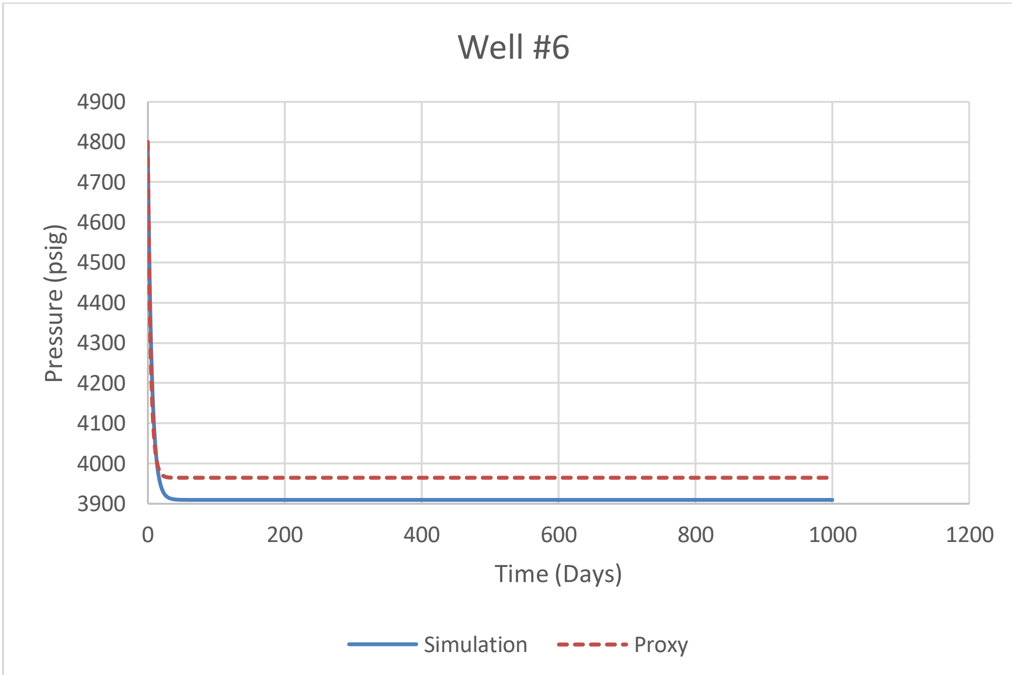

Figure 4.12: Pressure at well #6 for 3D Flow single rate

Table 4.9: Maximum error in each well for 3D Flow single rate case

	Well #1	Well #2	Well #3	Well #4	Well #5	Well #6
Max Error%	0.62	7.12	2.22	4.78	0.92	1.41

4.1.3.2 Multi Rate

The case study in this example is a single phase flow in three dimensions as the previous example, with the same permeability distribution and reservoir boundary conditions. The difference is that in this example there are seven wells located in the reservoir, four of them are producers and the other three are injectors as illustrated in Figure 4.13. In this case all wells operate at variable rates, as the well flow at a specific rate and then change to another value. The objective of this example is to illustrate the capability of the proxy model to capture the effect of changing the rate on the objective well and all other adjacent wells.

In this case, even though there is some difference between the two methods, the proxy model behave exactly as the numerical reservoir simulation following the same pattern in all the considered wells as in Figure 4.14 through Figure 4.20.

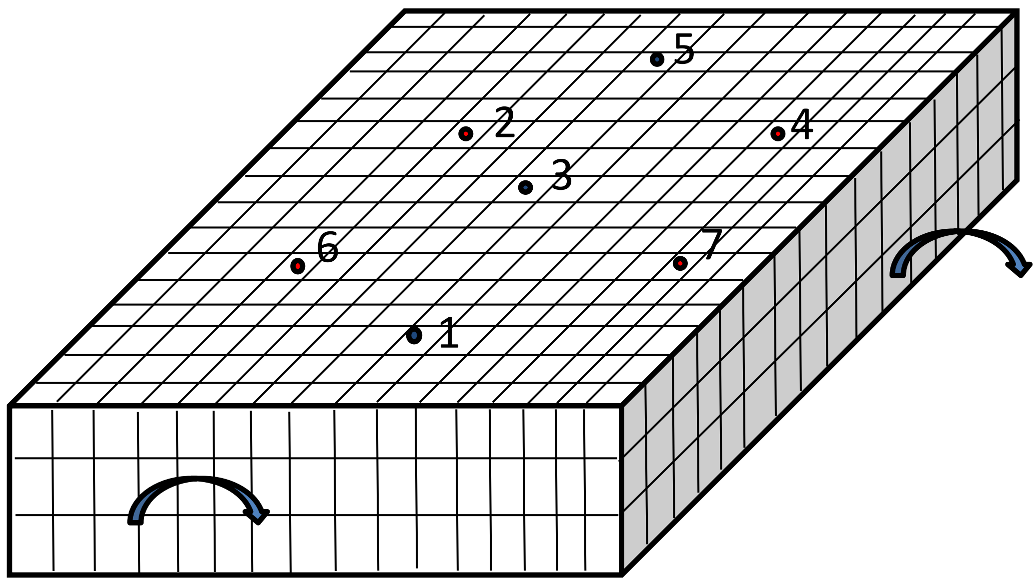

Figure 4.13: 3D Flow multi rate case

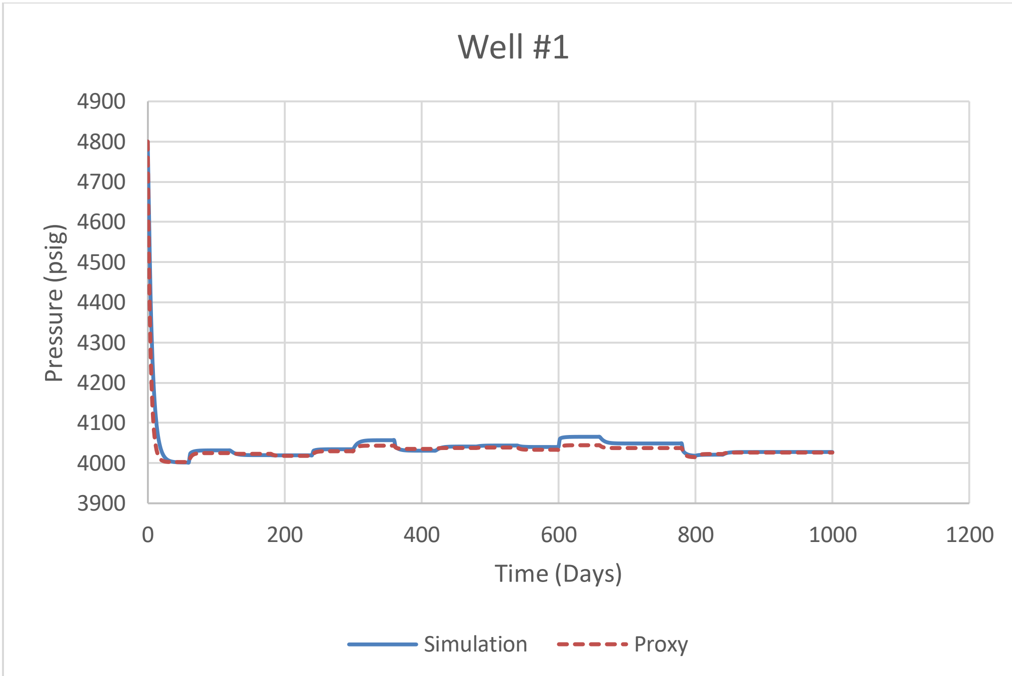

Figure 4.14: Pressure at well #1 for 3D Flow multi rate

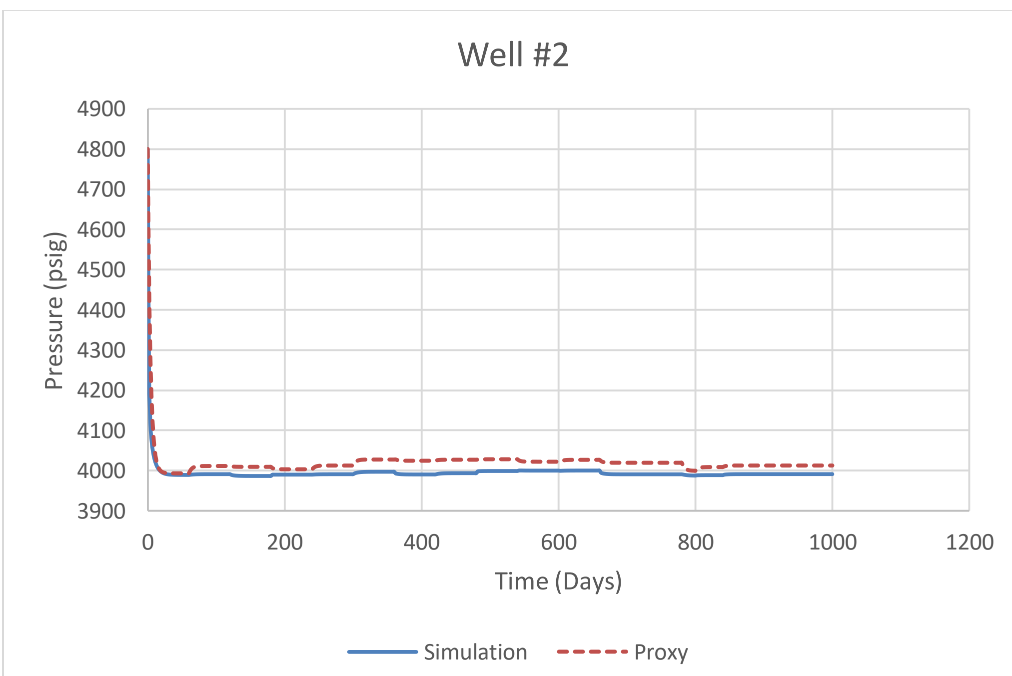

Figure 4.15: Pressure at well #2 for 3D Flow multi rate

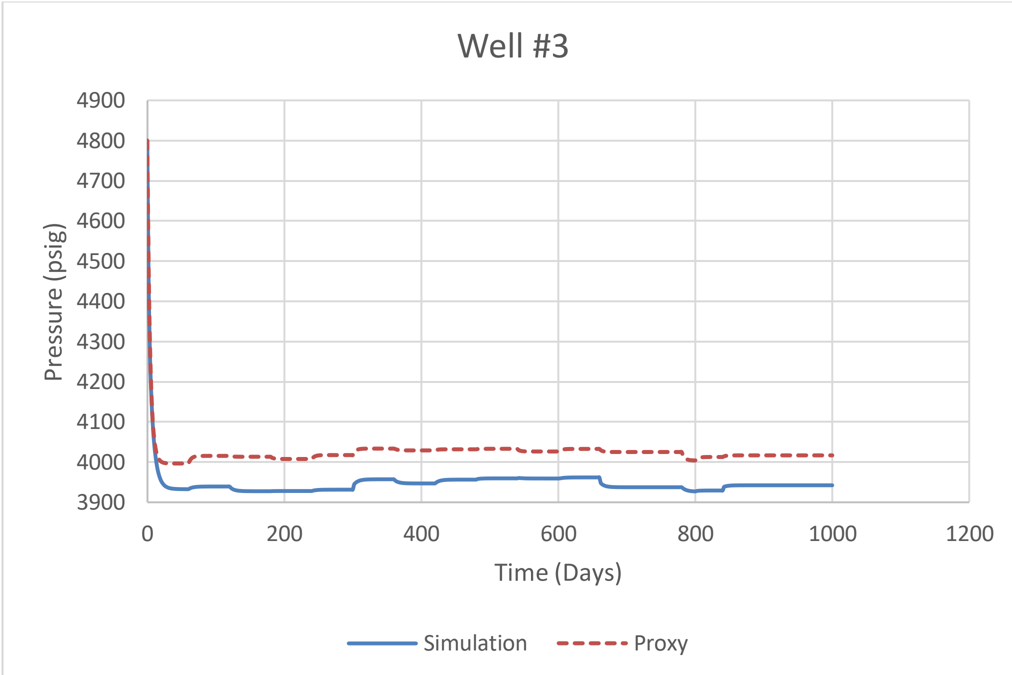

Figure 4.16: Pressure at well #3 for 3D Flow multi rate

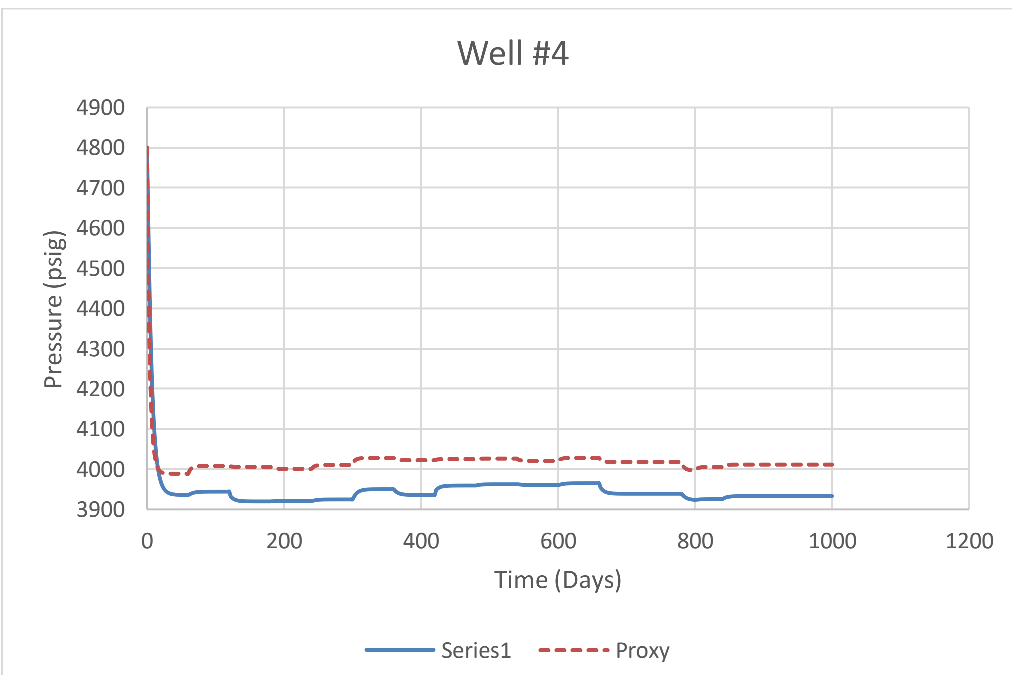

Figure 4.17: Pressure at well #4 for 3D Flow multi rate

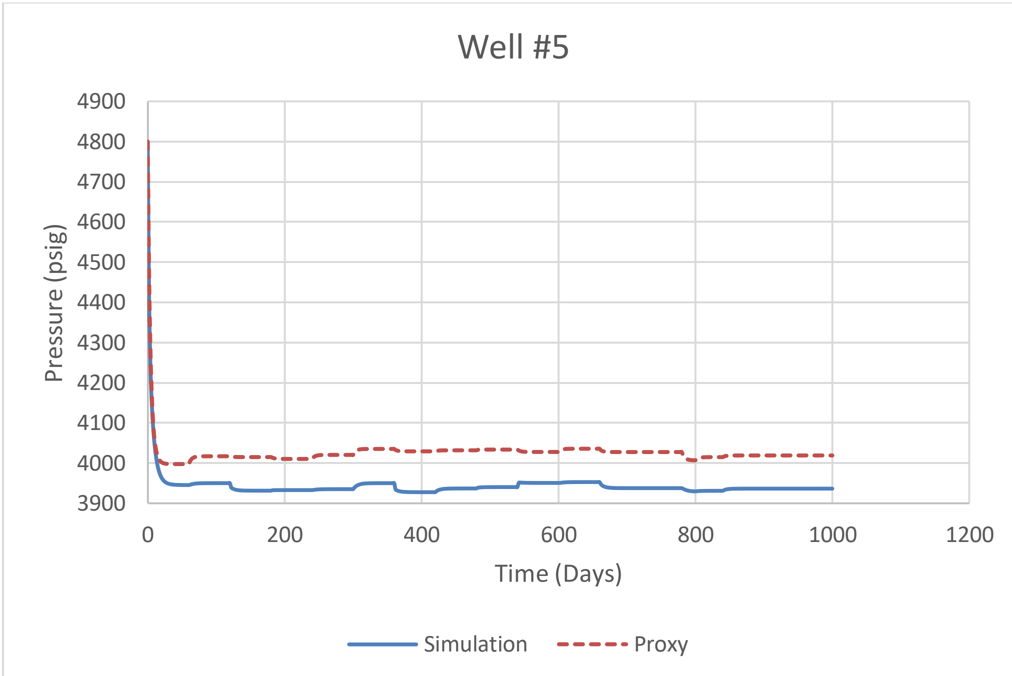

Figure 4.18: Pressure at well #5 for 3D Flow multi rate

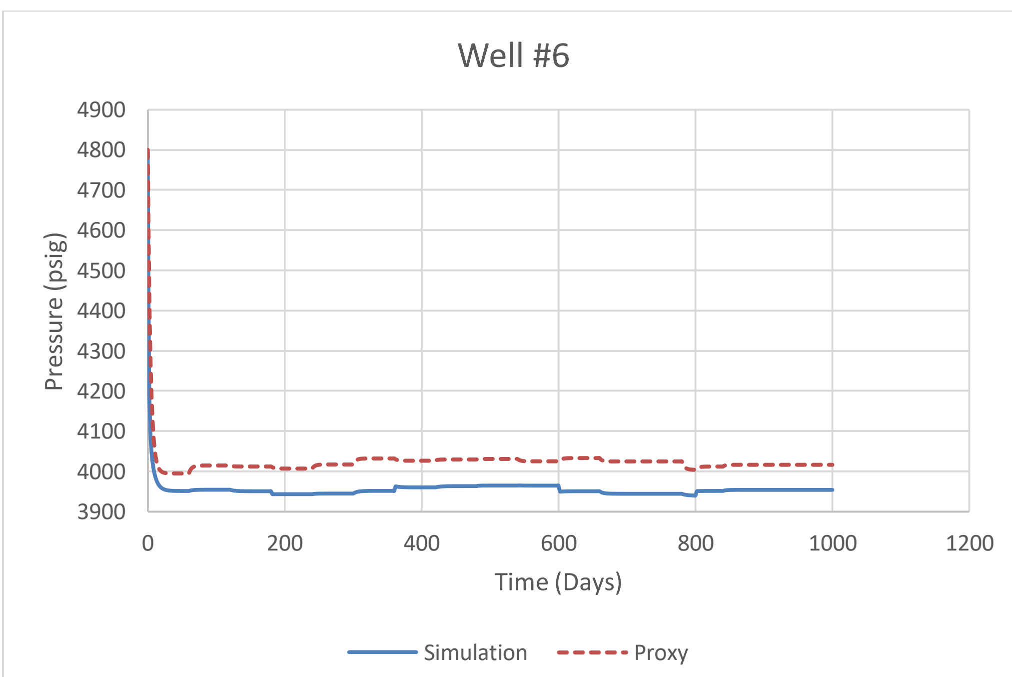

Figure 4.19: Pressure at well #6 for 3D Flow multi rate

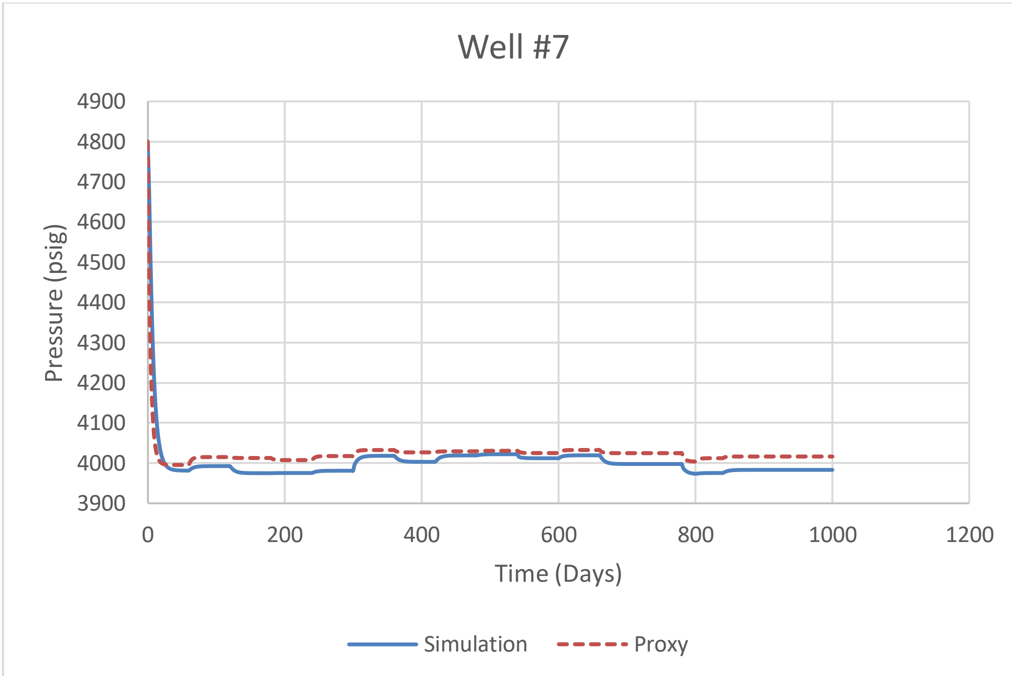

Figure 4.20: Pressure at well #7 for 3D Flow multi rate

Table 4.10: Maximum error in each well for 3D Flow multi rate case

	Well #1	Well #2	Well #3	Well #4	Well #5	Well #6	Well #7
Max Error%	3.05	5.43	2.23	3.27	2.59	5.47	5.22

4.2 Two Phase

The ‘Egg Model’ is a synthetic reservoir model consisting of an ensemble of 101 relatively small three-dimensional realizations of a channelized oil reservoir in the form of discrete permeability fields modelled with 60*60*7 = 25.200 grid cells of which 18.553 cells are active, the non-active cells lay outside of the model leaving an active egg shaped reservoir model. In most of the publications that used this model, it has been used to simulate two-phase flow of oil and water. Because it has no aquifer and no gas cap, and the primary production is negligible, the only production mechanism is water flooding with the aid of eight injection wells and four production wells. It has been used in numerous publications to test algorithms for computer-assisted flooding optimization, history matching or, in combination, closed-loop reservoir management. Unfortunately, the details of the model parameters settings are not always the same and not always fully documented in most of the publications. The channels with high-permeability values in a low-permeable background demonstrate a meandering river patterns as in fluvial environments, Figure 4.21 shows the permeability distribution with the well placement in the model.

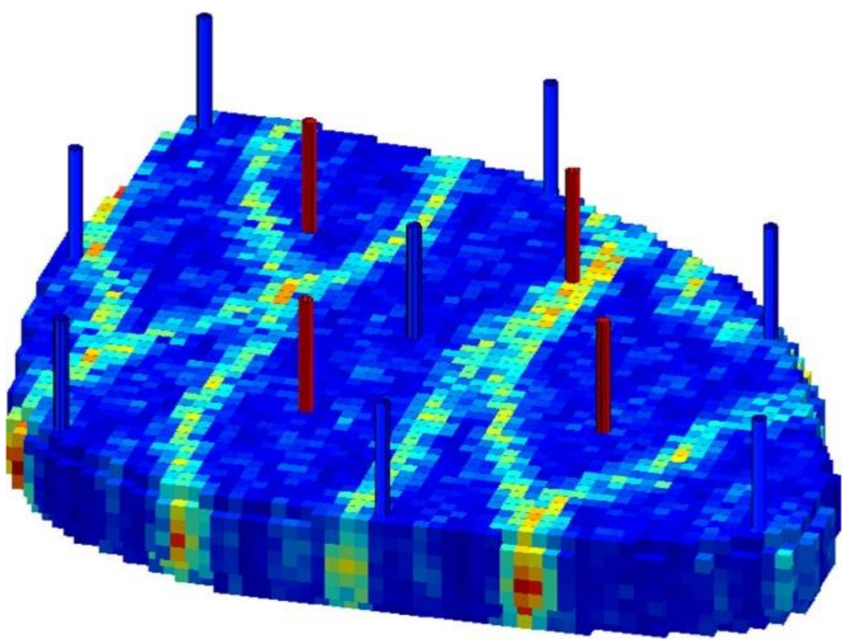

Figure 4.21: Egg model

For this example, only the first top layer of the model was considered with the same permeability distribution and well placement. Two cases with different injection rates were studied and compared. The following section illustrate the results obtained from the two cases, and the comparison between the numerical reservoir simulation and the proposed proxy model. The model parameters are listed below in Table 4.11: Egg model parameters.

Table 4.11: Egg model parameters

Symbol	Variable	Value	Field Units
h	Grid block height	13	ft
Δx, Δy	Grid block length / width	26	ft
Ø	Porosity	0.2	
c_o	Oil compressibility	$1.2*10^{-5}$	Psi^{-1}
c_r	Rock compressibility	0	Psi^{-1}
c_w	Water compressibility	$1*10^{-6}$	Psi^{-1}
μ_o	Oil dynamic viscosity	5	cp
μ_w	Water dynamic viscosity	1	cp
k_{ro}^0	End-point relative permeability of oil	0.8	
k_{rw}^0	End-point relative permeability of water	0.75	
n_o	Corey model exponent of oil	4	
n_w	Corey model exponent of water	3	
S_{or}	Residual oil saturation	0.1	
S_{wc}	Connate water saturation	0.1	
p_c	Capillary pressure	0	psi
p_i	Initial reservoir pressure (top layer)	5800	psig
S_{wi}	Initial water saturation	0.2	
q_{wi}	Water injection rates, per well	250/150	STB/D
p_{bh}	Production well bottom-hole pressure	5700	psig
r_w	Well-bore radius	0.3	ft
T	Simulation time	3600	Day

4.2.1 Case 1

In this case the injection rate selected was 250 STB/D with a maximum injection pressure of 6900 psig. The oil production rate estimated using the proxy model for producer #1, 3# and #4 is higher than that of the numerical simulation, while producer #2 is exactly the opposite. The same thing goes with the water production rate, but in the two cases the proxy model is following the same trend of the numerical simulation. Table 4.12 shows the values estimated for the model parameters in this case, and Figure 4.22 through Figure 4.29 show the comparison between the results obtained.

Table 4.12: Model parameters for two phase Flow

a1	a2	a3	a4	a5	a6	a7	a8	a9	a10
1.21	0.99	0.93	1.37	0.98	1.23	0.96	1.23	0.85	0.85

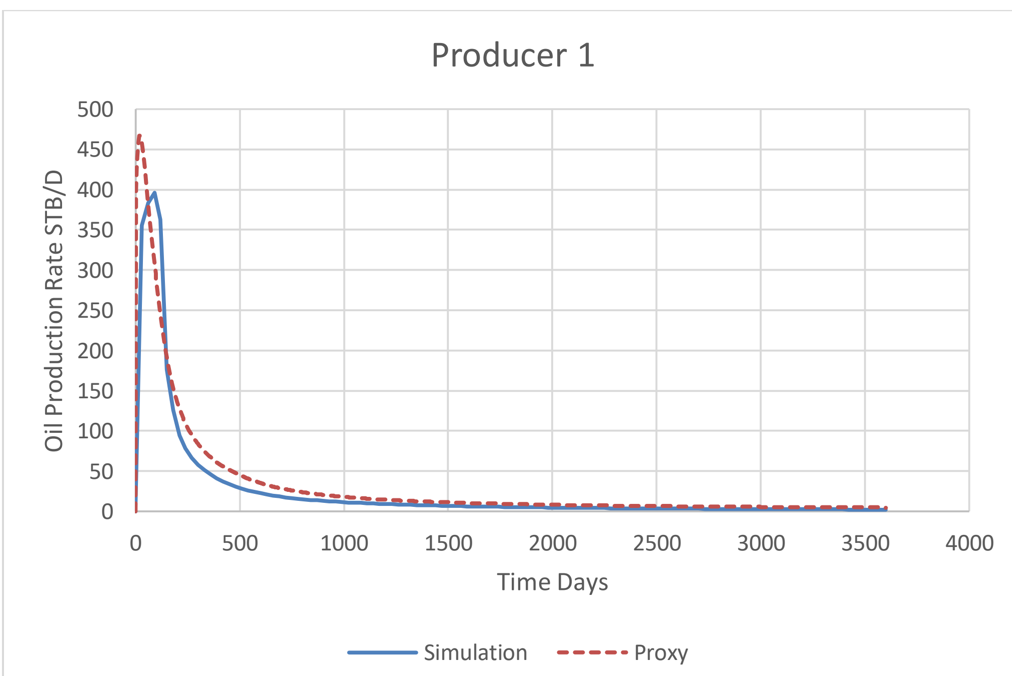

Figure 4.22: Two phase Case 1 Producer #1 Oil production rate

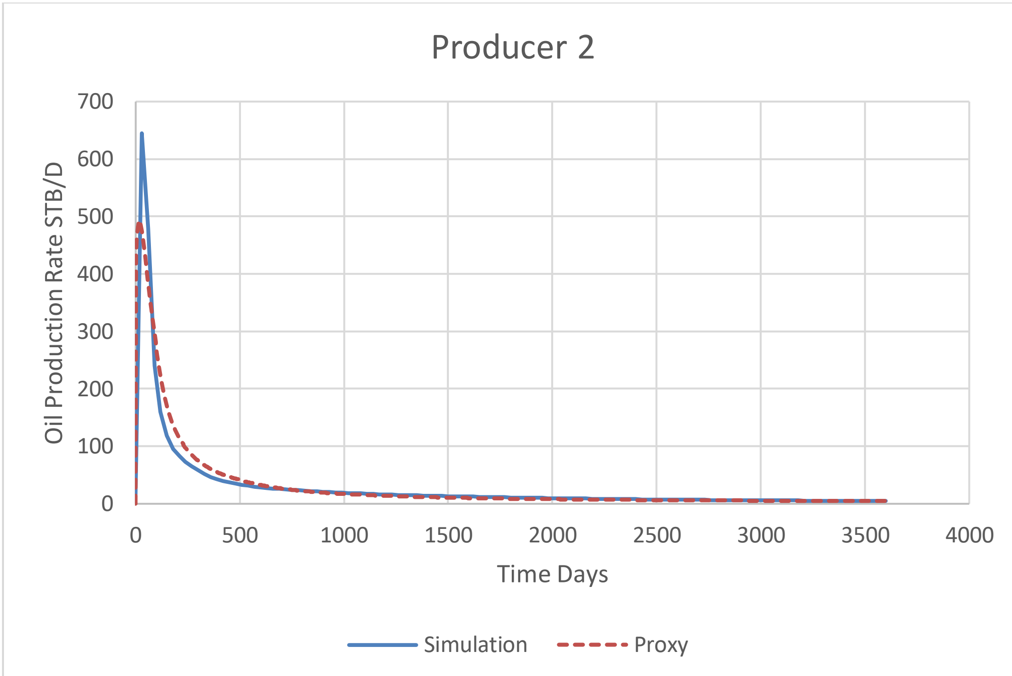

Figure 4.23: Two phase Case 1 Producer #2 Oil production rate

Figure 4.24: Two phase Case 1 Producer #3 Oil production rate

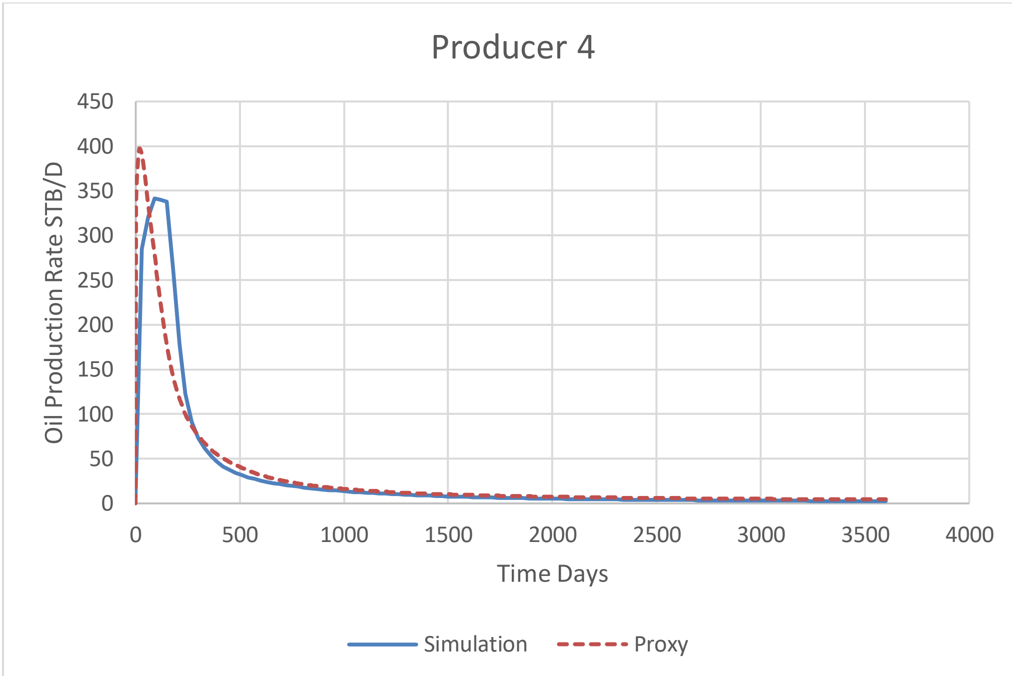

Figure 4.25: Two phase Case 1 Producer #4 Oil production rate

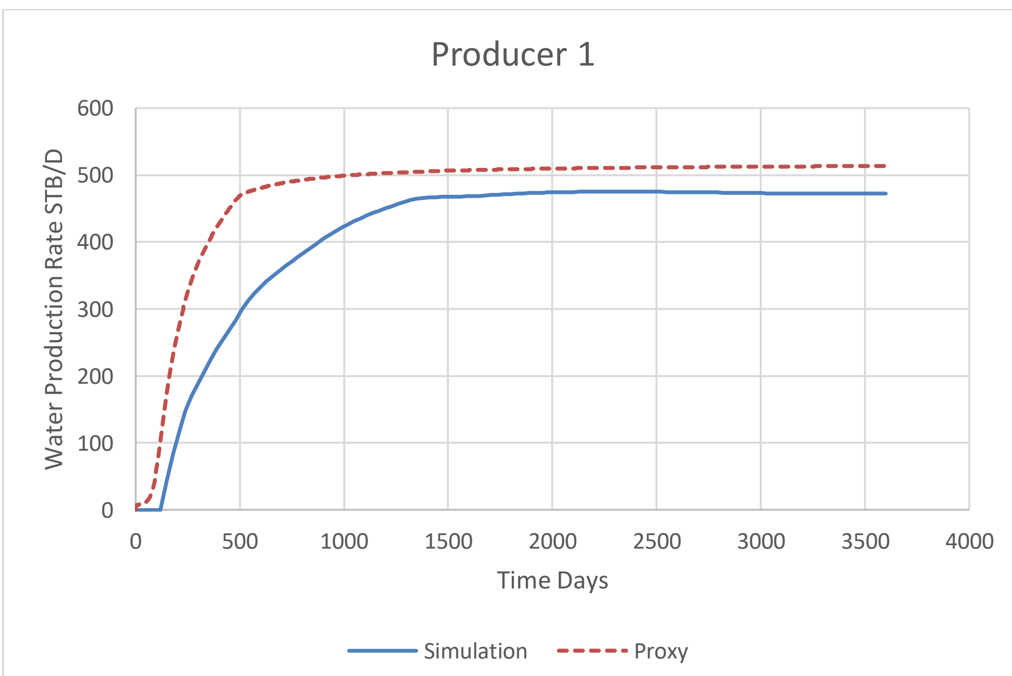

Figure 4.26: Two phase Case 1 Producer #1 Water production rate

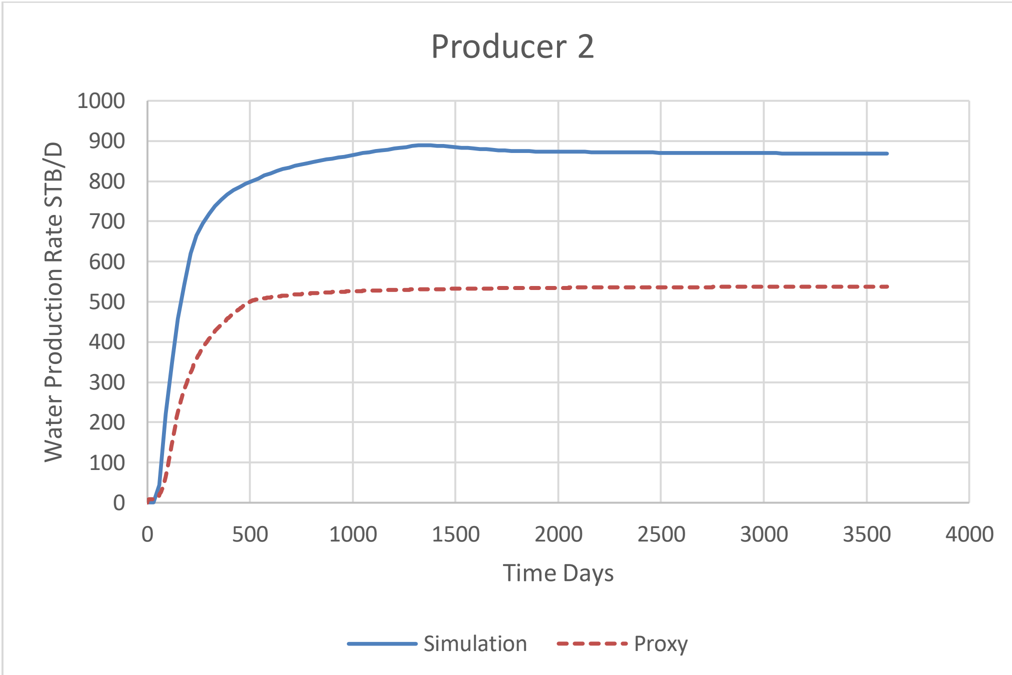

Figure 4.27: Two phase Case 1 Producer #2 Water production rate

Figure 4.28: Two phase Case 1 Producer #3 Water production rate

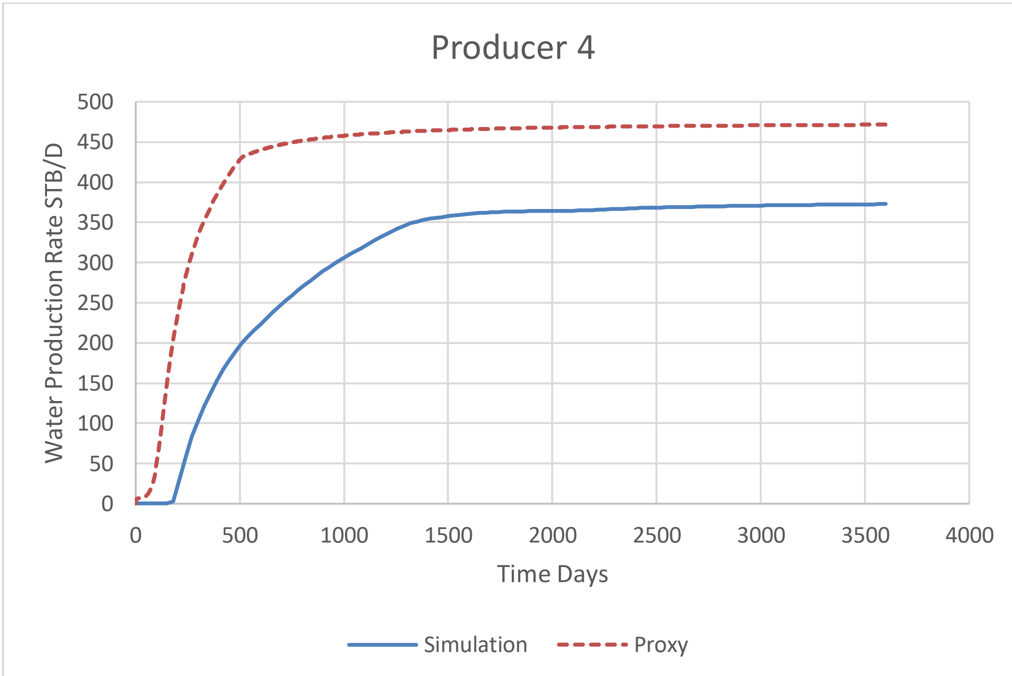

Figure 4.29: Two phase Case 1 Producer #4 Water production rate

Considering the total field production rate for both oil and water and the field injection rate, the results obtained from the proxy model were very close to those of the numerical reservoir simulation as displayed in Figure 4.30 through Figure 4.32.

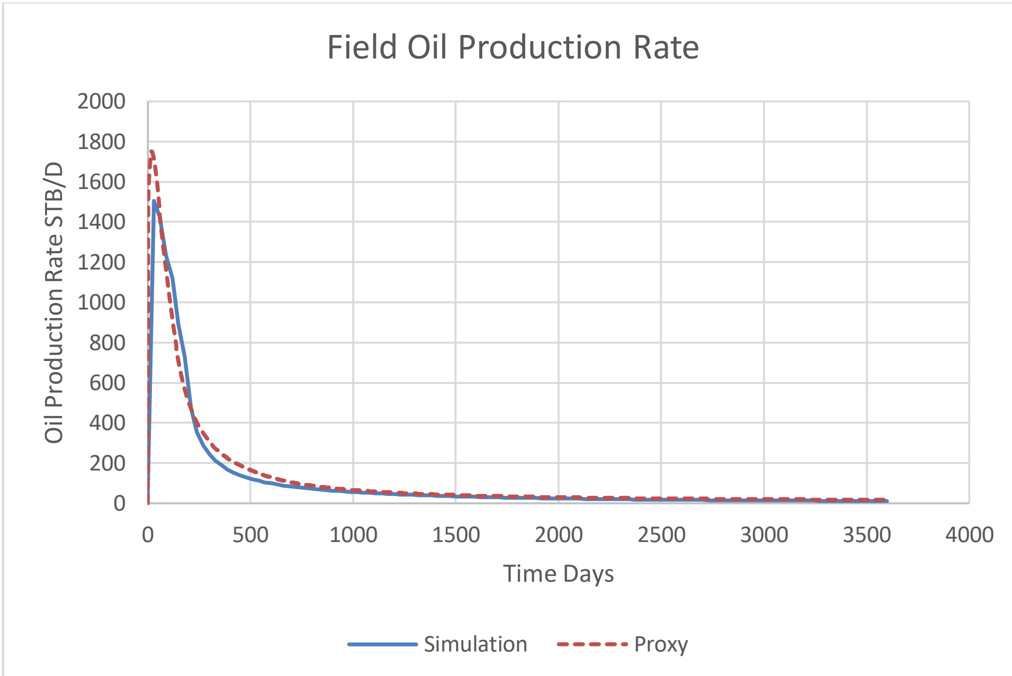

Figure 4.30: Two phase Case 1 Field oil production rate

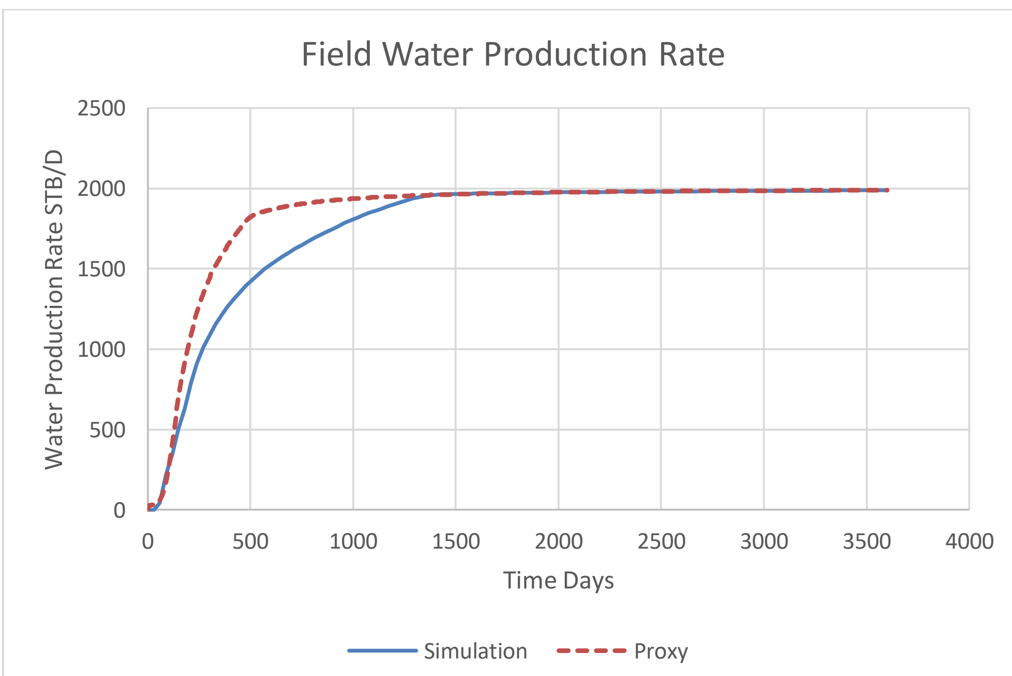

Figure 4.31: Two phase Case 1 Field water production rate

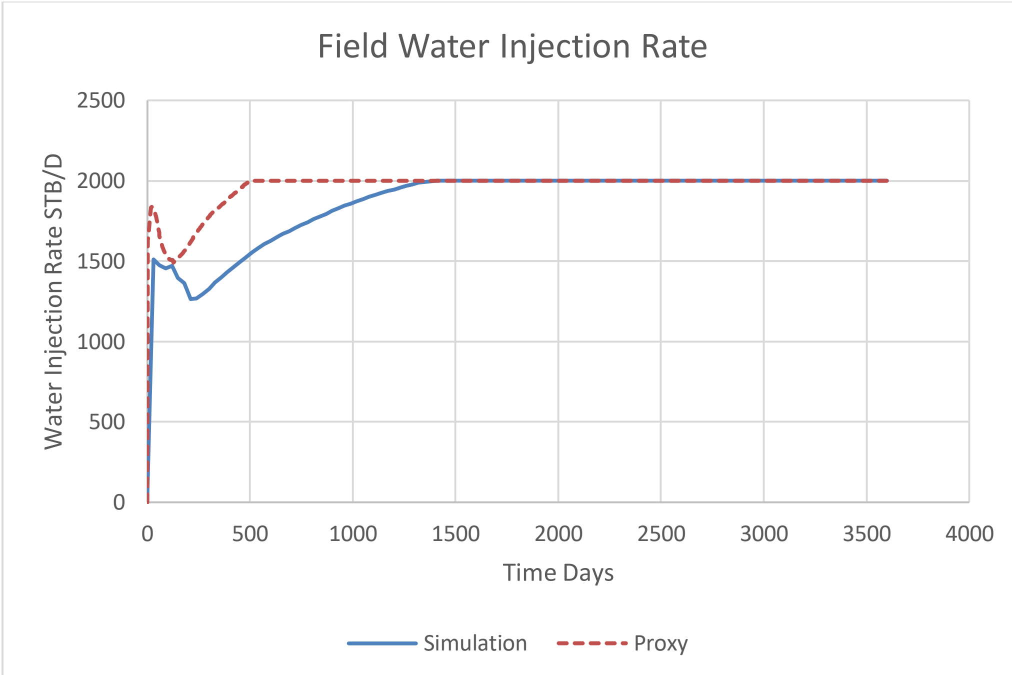

Figure 4.32: Two phase Case 1 Field water injection rate

At the end of the case, the cumulative oil production, water production and volume of water injected were calculated for this case from both the numerical simulation and the proxy model. The results for the three parameters were close from the two methods with an error of 13.6%, 3.6% and 4.7% respectively. The elapsed time ratio between the numerical reservoir simulation and the proxy model ~32 times faster.

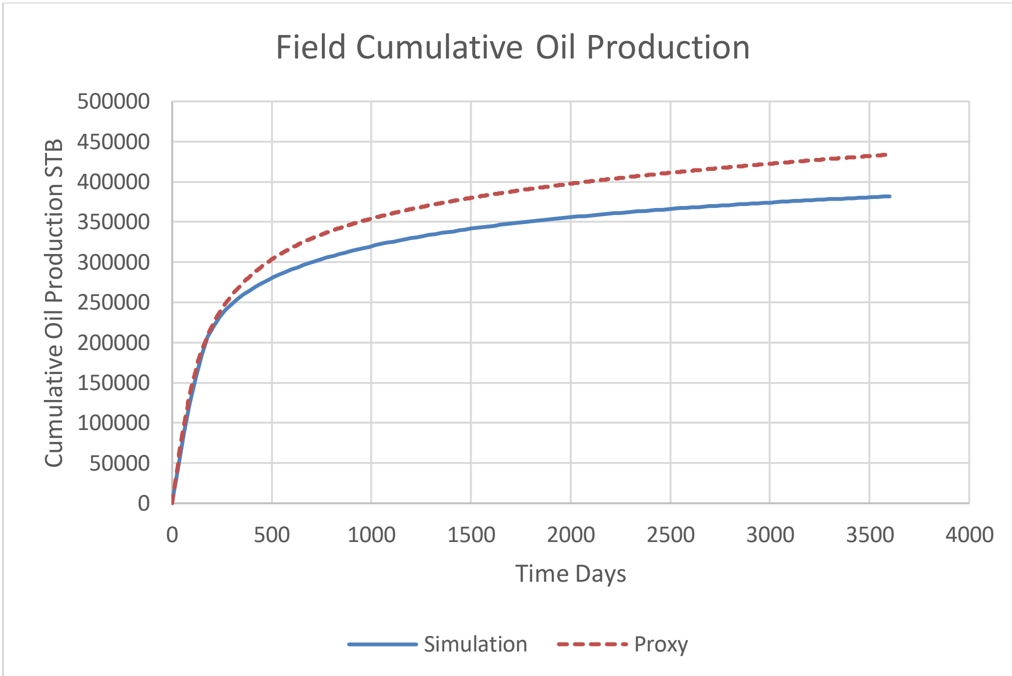

Figure 4.33: Two phase Case 1 Field oil cumulative production

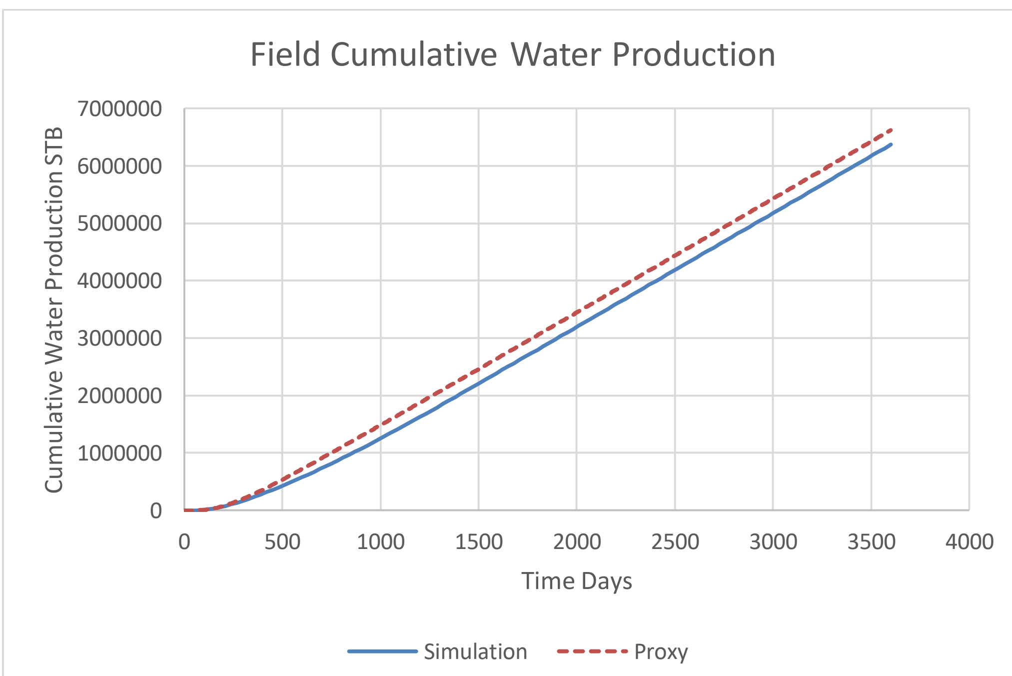

Figure 4.34: Two phase Case 1 Field water cumulative production

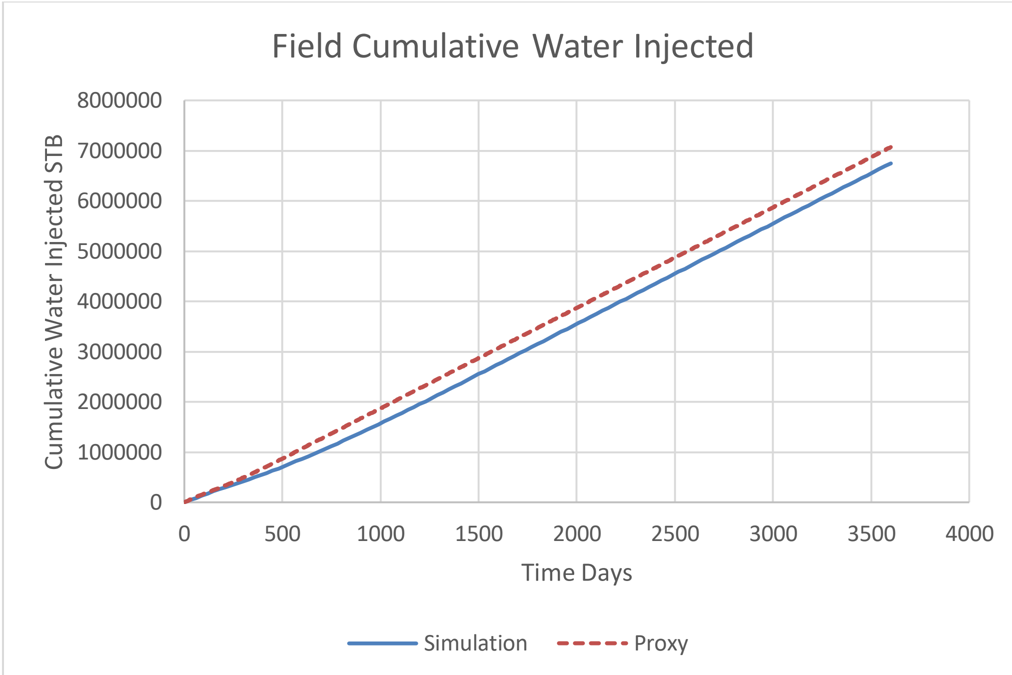

Figure 4.35: Two phase Case 1 Field cumulative water injection

4.2.2 Case 2

In this case the injection rate was reduced to 150 STB/D with the same maximum injection pressure of 6900 psig. Since the same reservoir and fluid properties remain the same, the model parameters estimated in Case 1 were used to simulate this case. Again, in this case the proxy model is following the same pattern of the numerical reservoir simulation. A comparison between the results is illustrated in Figure 4.36 through Figure 4.43.

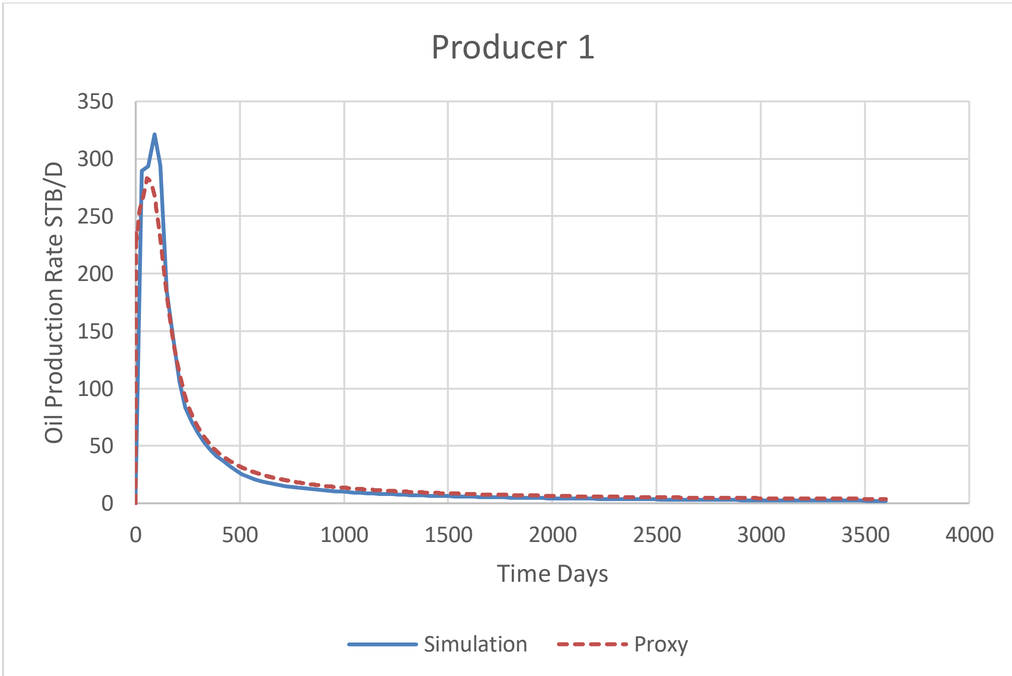

Figure 4.36: Two phase Case 2 Producer #1 Oil production rate

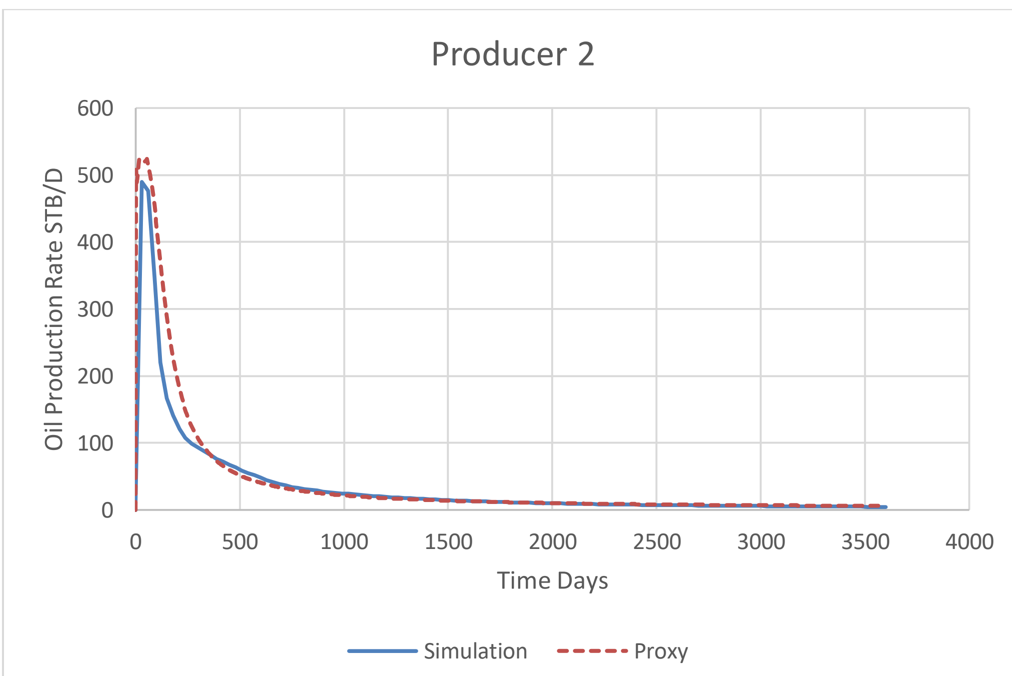

Figure 4.37: Two phase Case 2 Producer #2 Oil production rate

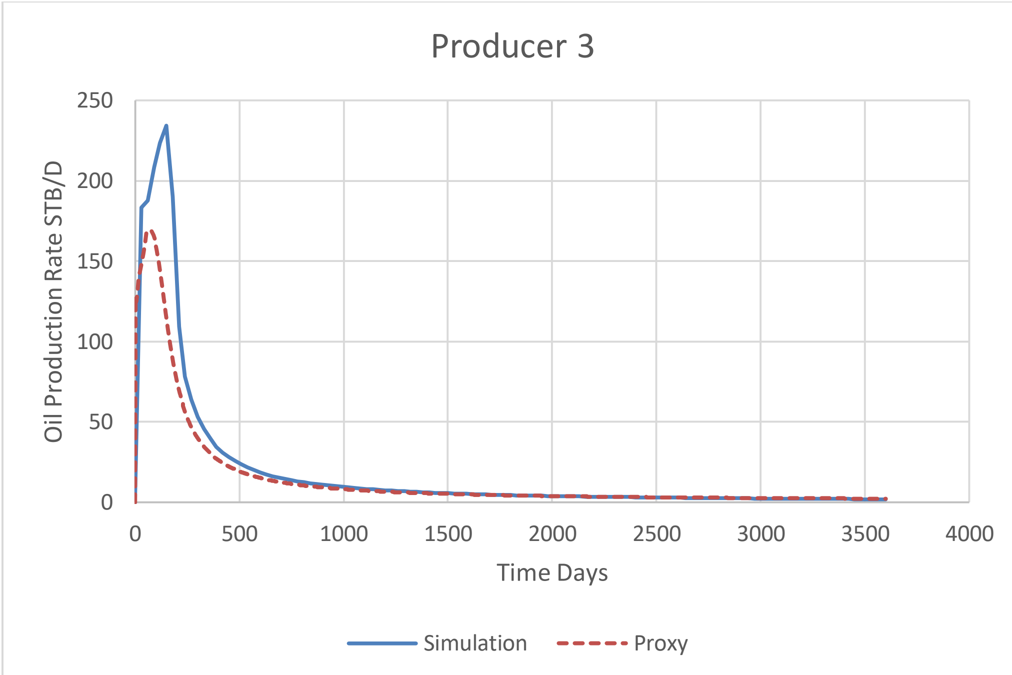

Figure 4.38: Two phase Case 2 Producer #3 Oil production rate

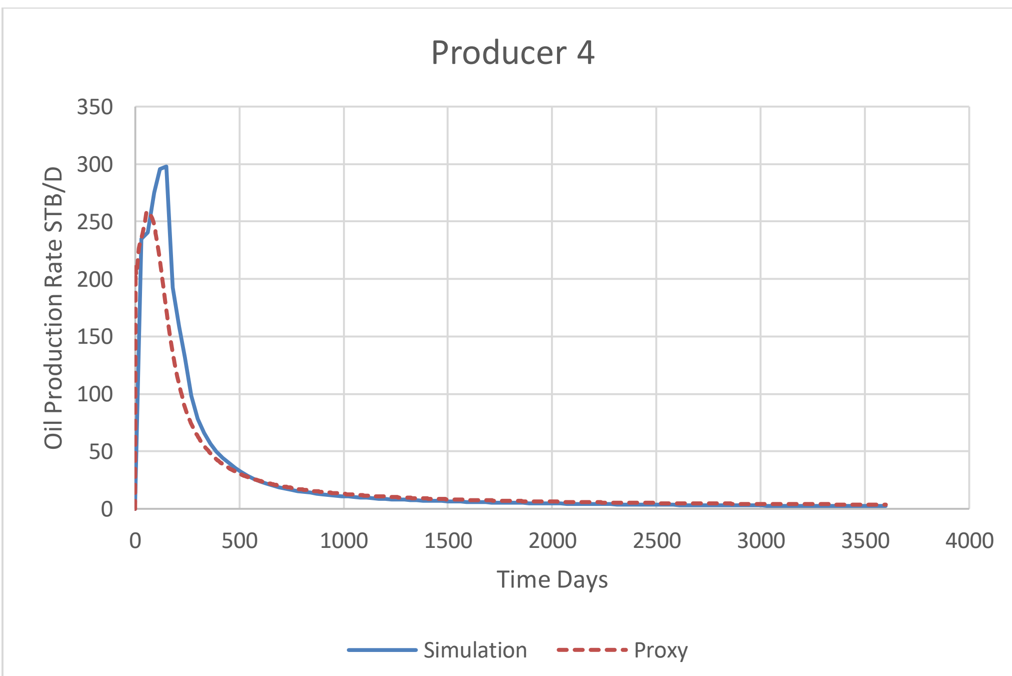

Figure 4.39: Two phase Case 2 Producer #4 Oil production rate

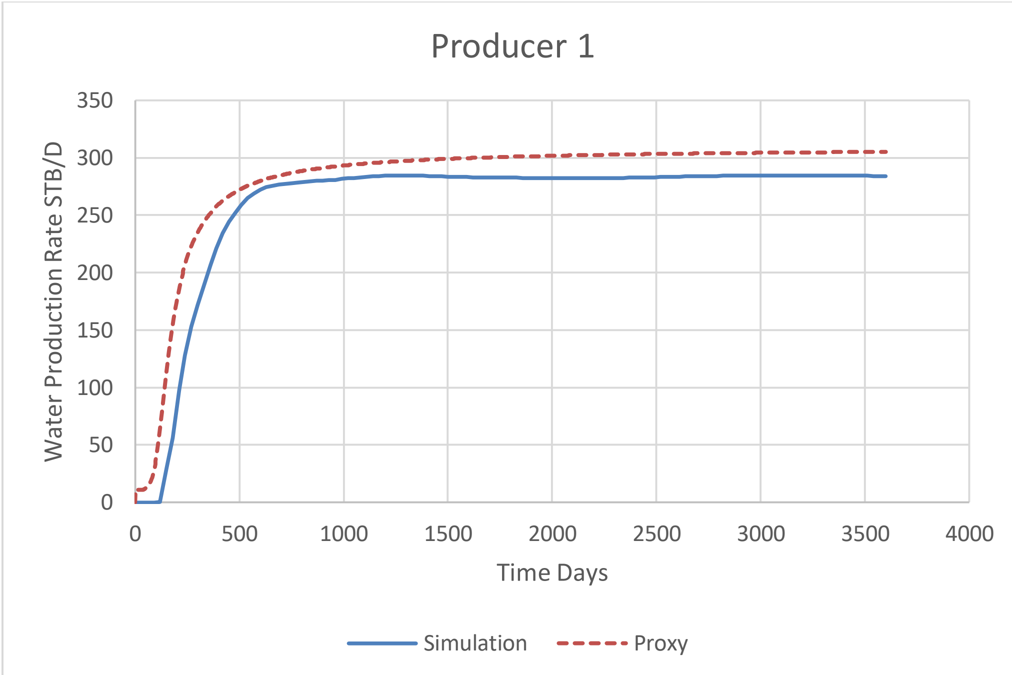

Figure 4.40: Two phase Case 2 Producer #1 Water production rate

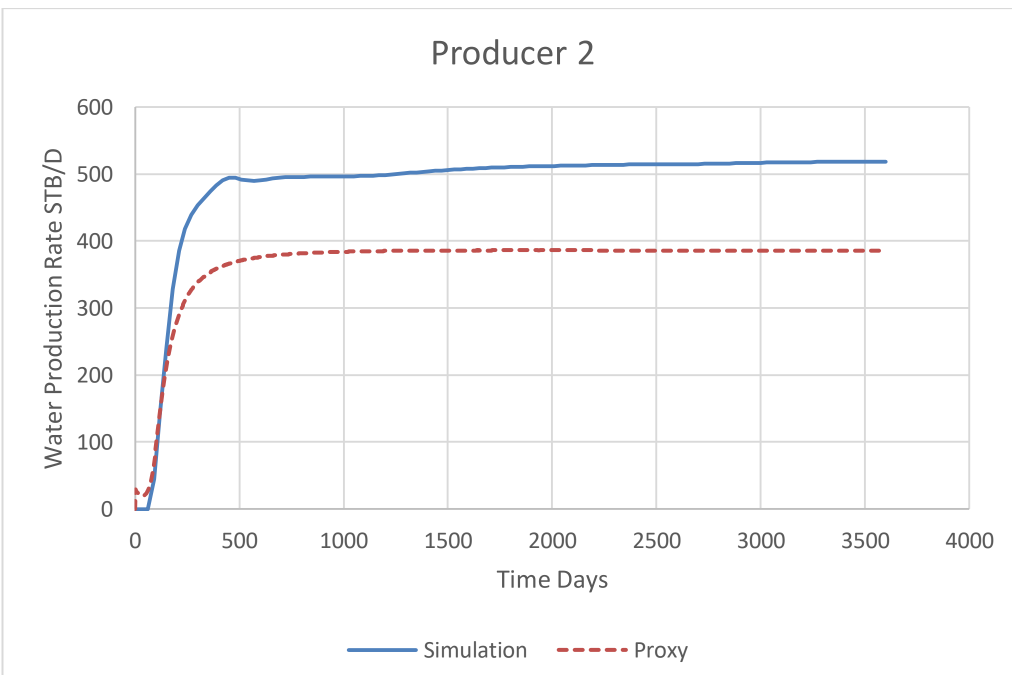

Figure 4.41: Two phase Case 2 Producer #2 Water production rate

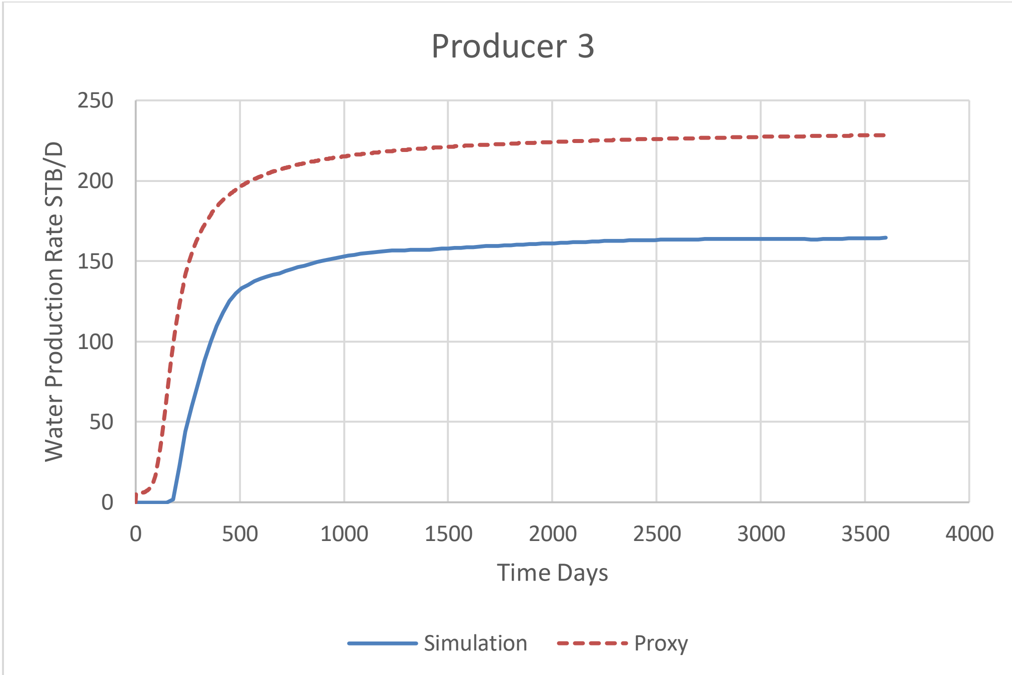

Figure 4.42: Two phase Case 2 Producer #3 Water production rate

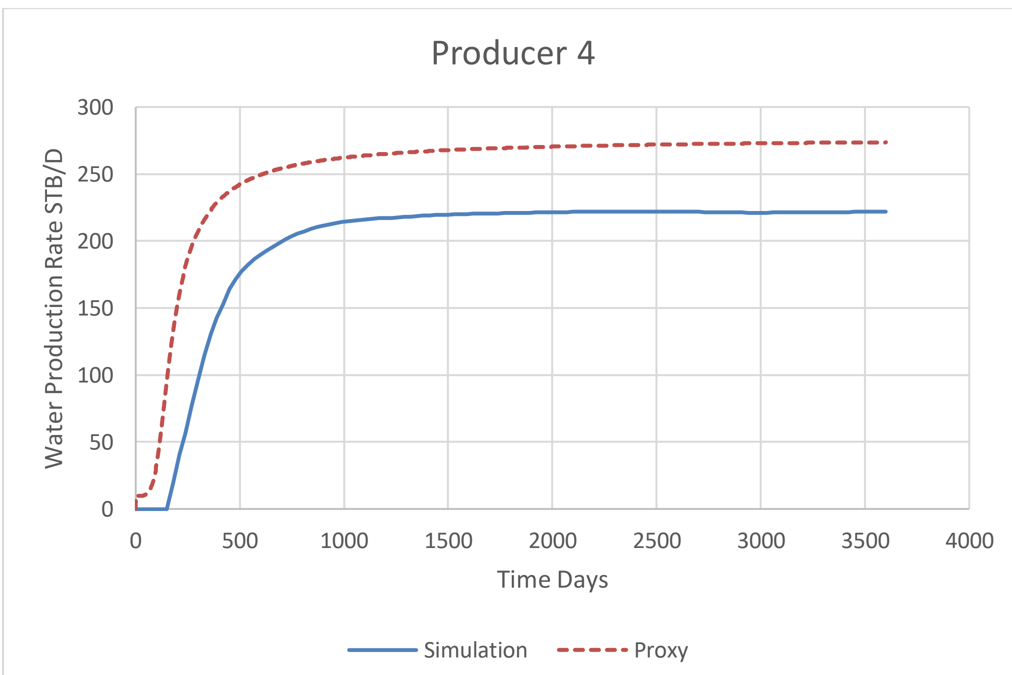

Figure 4.43: Two phase Case 2 Producer #4 Water production rate

As performed in case 1, the results for the total field production rate for both oil and water and the field injection rate were also compared. In this case it shows even a closer match than what was obtained in case 1 for the three parameters.

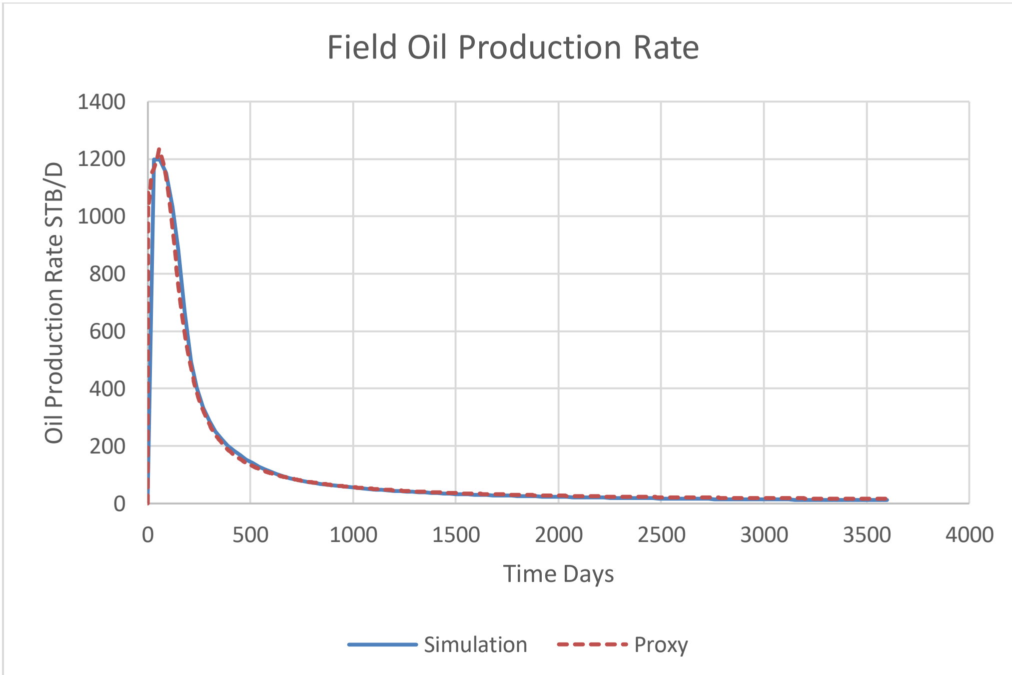

Figure 4.44: Two phase Case 2 Field oil production rate

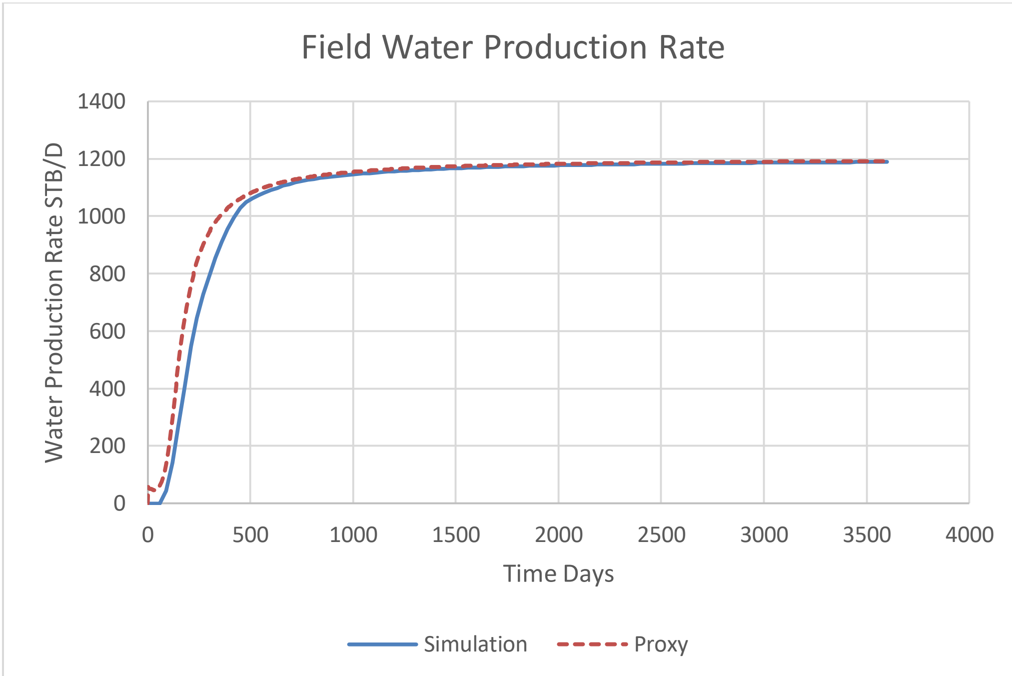

Figure 4.45: Two phase Case 2 Field water production rate

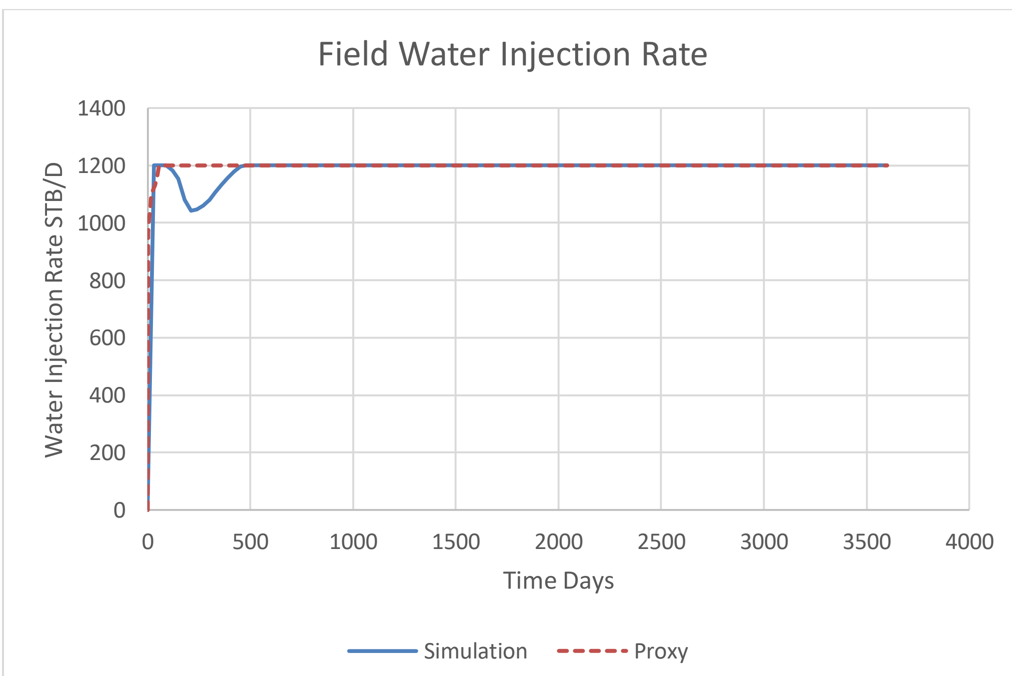

Figure 4.46: Two phase Case 2 Field water injection rate

Now looking at the cumulative oil production, water production and volume of water injected for both methods, the results are very close to each other with 2.9%, 1.6% and 0.5% error for cumulative oil production, water production and volume of water injected. It is showing the same pattern in both methods, again the elapsed time ratio between the numerical reservoir simulation and the proxy model ~32 times faster.

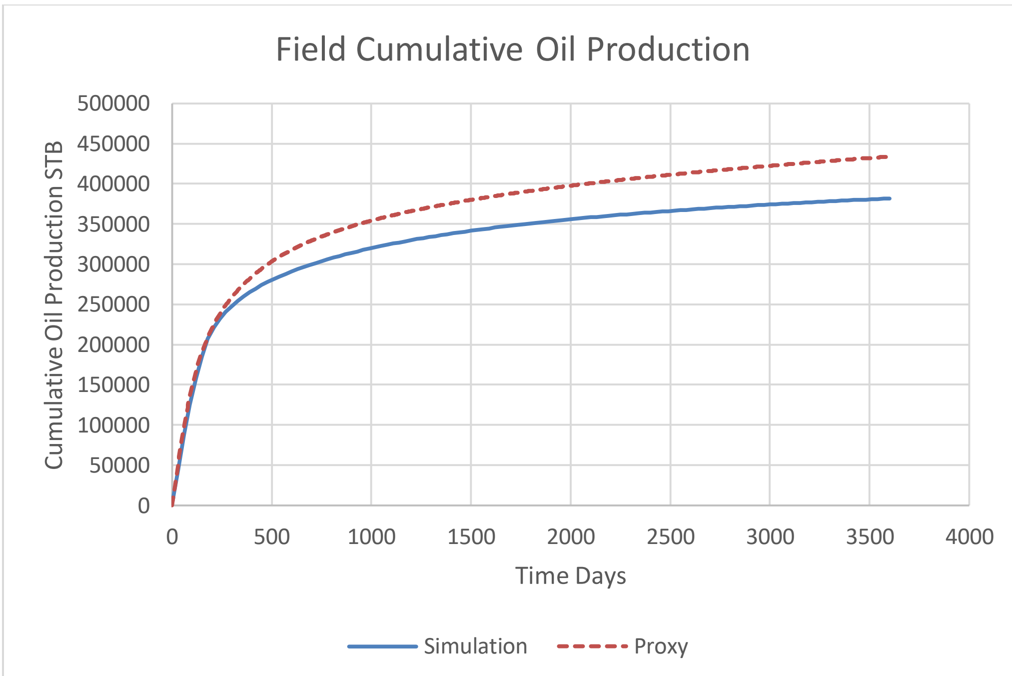

Figure 4.47: Two phase Case 2 Field oil cumulative production

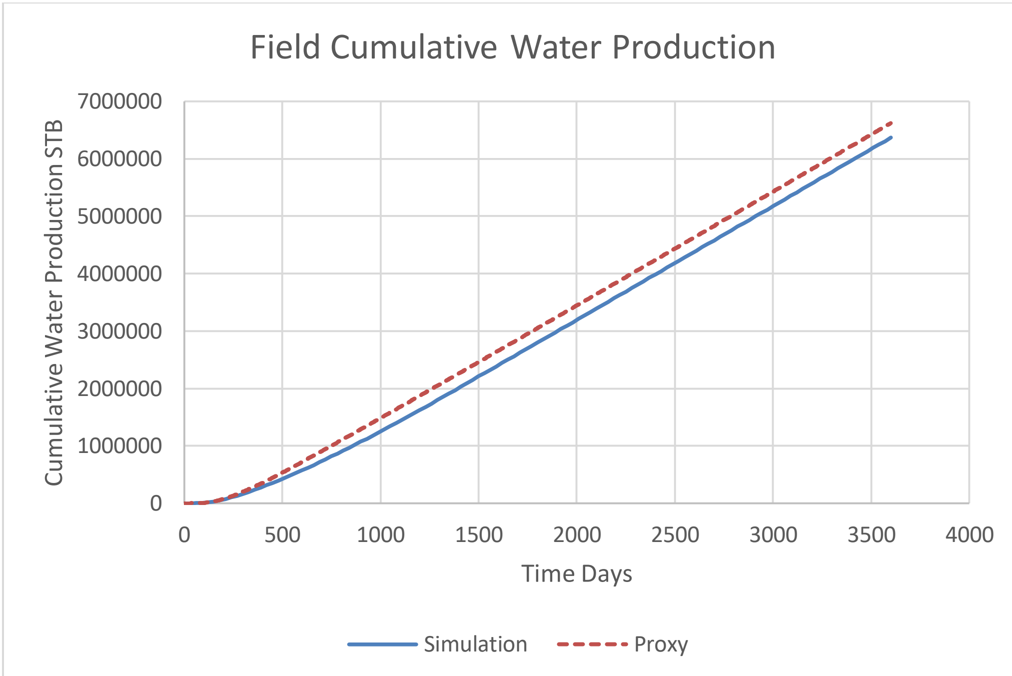

Figure 4.48: Two phase Case 2 Field water cumulative production

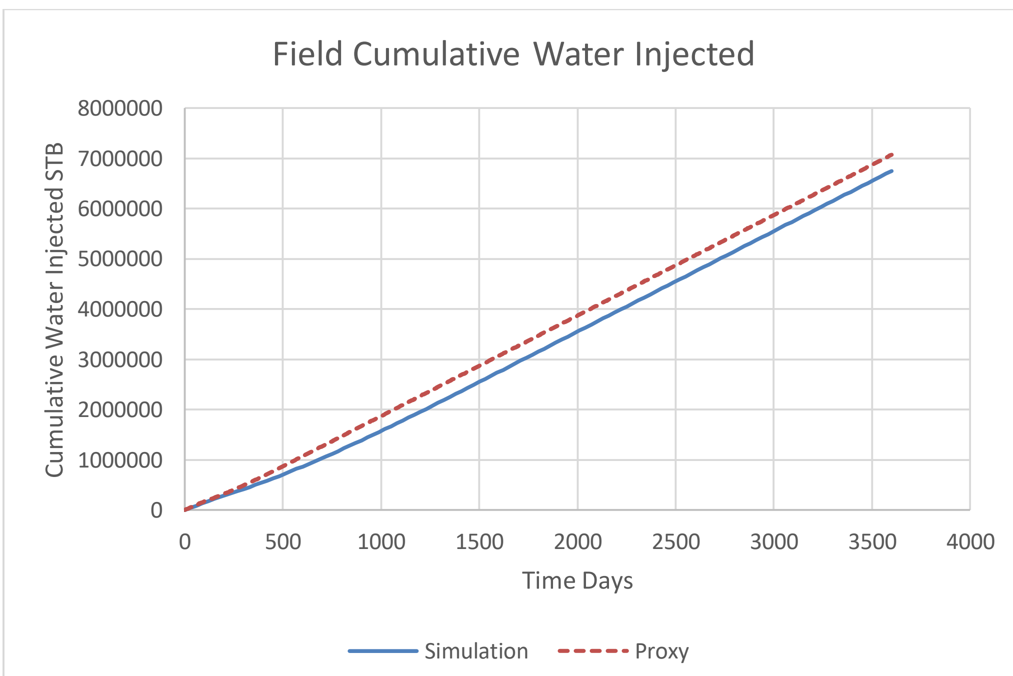

Figure 4.49: Two phase Case 2 Field cumulative water injection

4.3 Material Balance

Considering Case 1 and Case 2 in the Two Phase section, the applicability of the material balance on the proxy model was tested to check if the proxy model honors the overall material balance of the system. As the production is only due to the water injection, and no influx or out flux at the boundaries, the volume of the total production should be equal to the total volume of water injected at any given time. By selecting random points from Case 2, the overall material balance was checked. As shown in Table 4.13, it can be said that the proxy model honors the material balance.

Table 4.13: Material Balance Check

Time Days	**Water Injection Rate STB/D**	**Water Production Rate STB/D**	**Oil Production Rate STB/D**	**Mass Balance** $mass_{in}$ - $mass_{out}$
1197	**1200**	**1141.85**	**58.87**	**-0.72 ≈ 0**
2997	**1200**	**1179.19**	**21.05**	**-0.24 ≈ 0**
3600	**1200**	**1182.99**	**17.19**	**-0.18 ≈ 0**

4.4 Developmental Study

At this point the objective is to select what is the better case scenario between the two cases. The comparison was performed on the data obtained by the numerical simulation, and the on the results of the proxy model separately. As illustrated in the figures from Figure 4.50 to Figure 4.55, both methods showed a higher value in the cumulative oil production, water production and volume of water injected for case 1. This result show that the proxy model is capable to perform developmental studies and conduct comparison between different case scenarios in order to identify the best scenario.

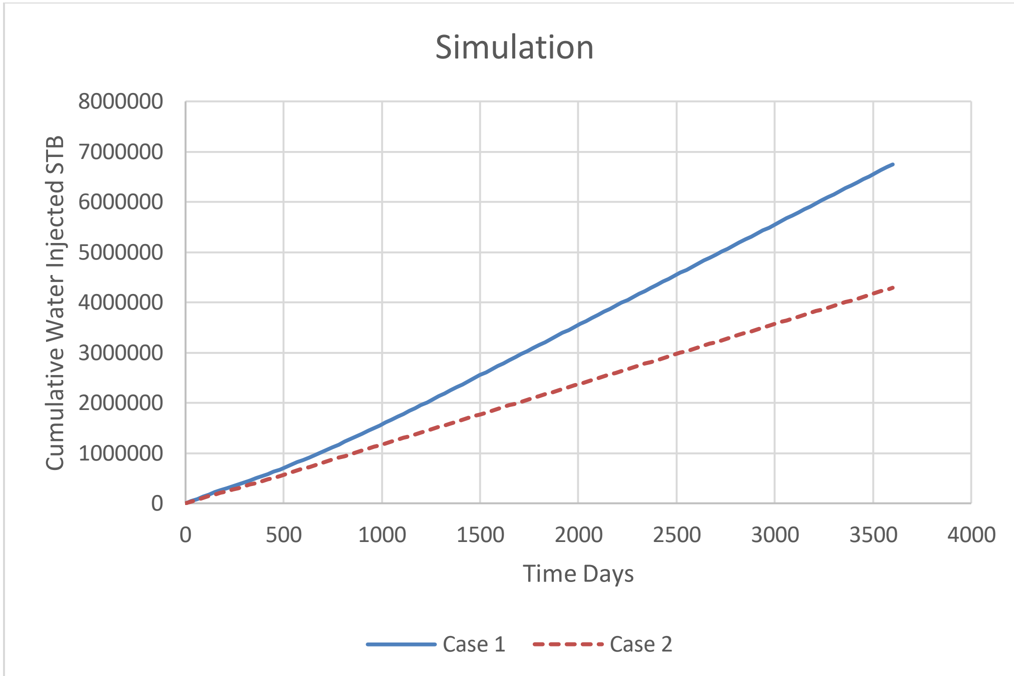

Figure 4.50: Numerical simulation cumulative water injection Case 1, Case 2

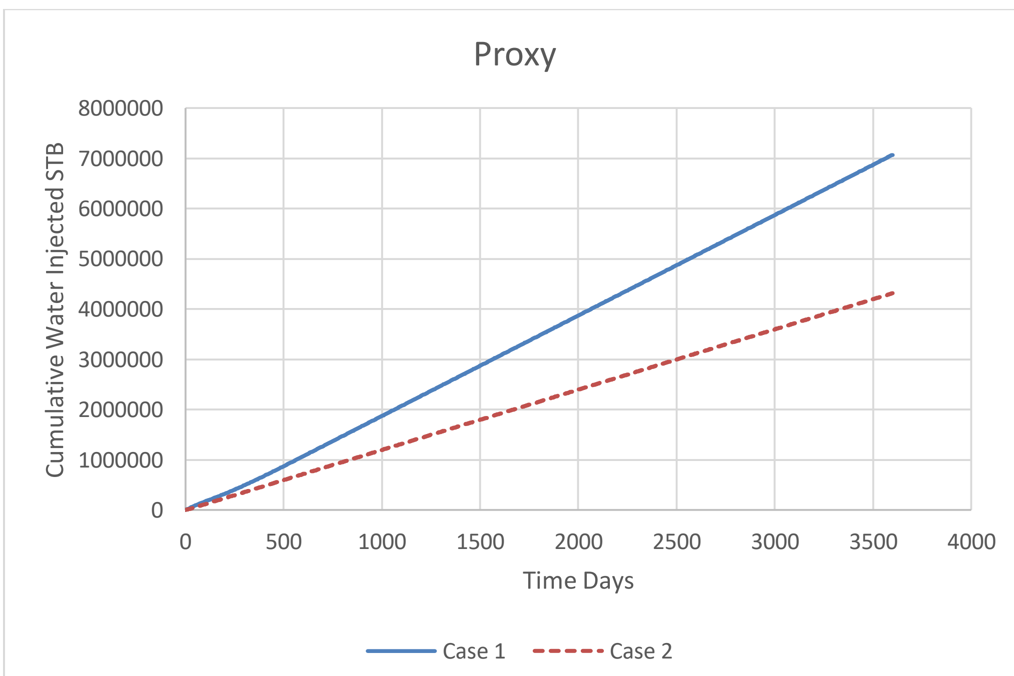

Figure 4.51: Proxy model cumulative water injection Case 1, Case 2

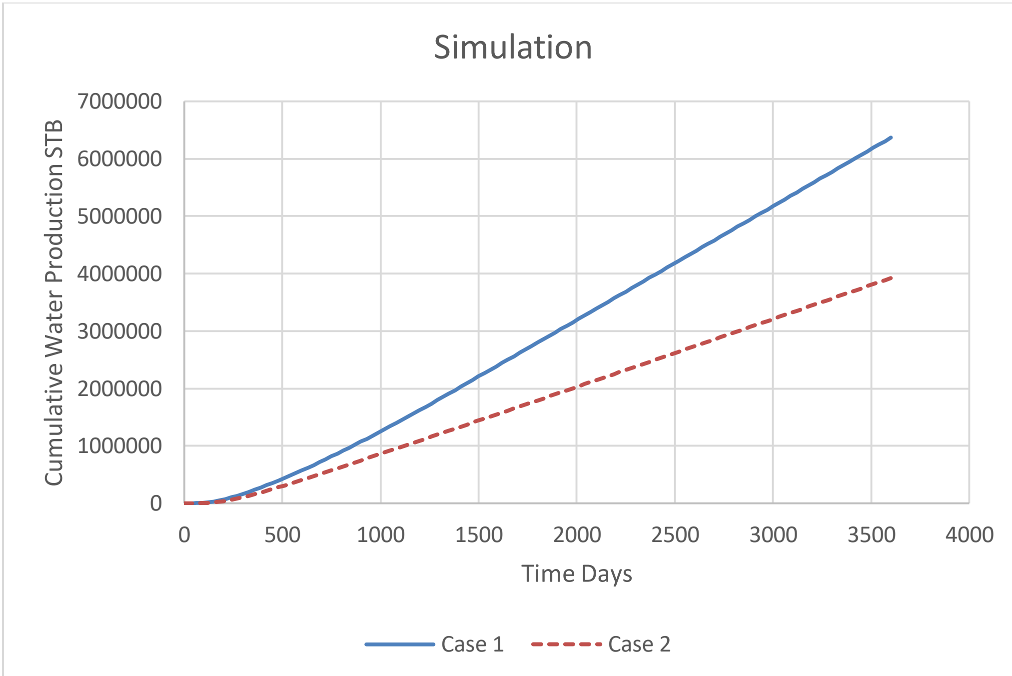

Figure 4.52: Numerical simulation cumulative water production Case 1, Case 2

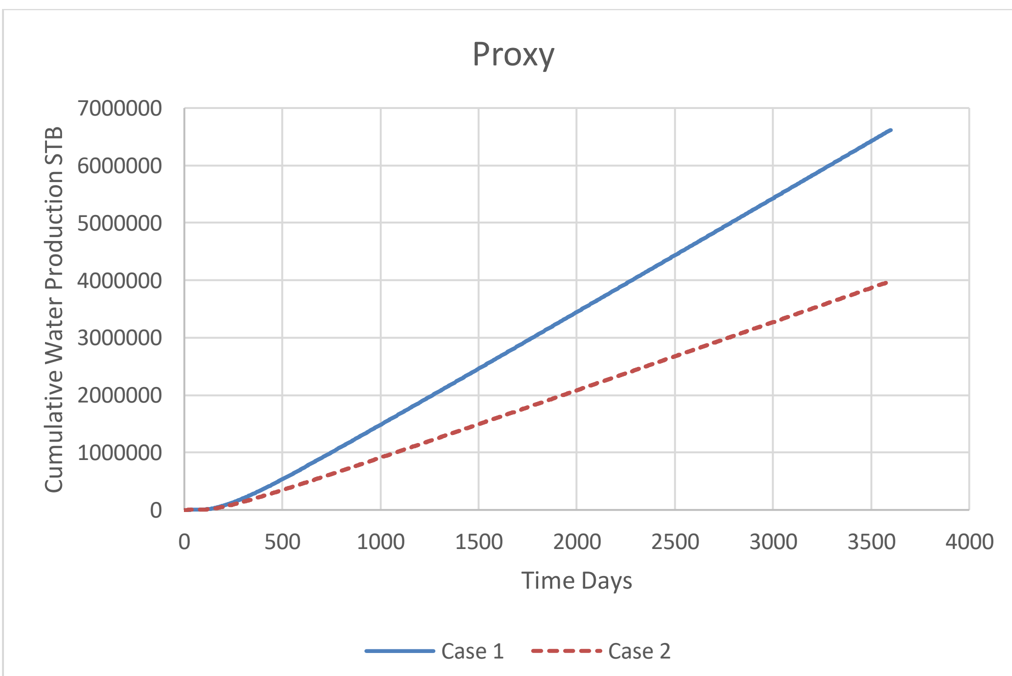

Figure 4.53: Proxy model cumulative water production Case 1, Case 2

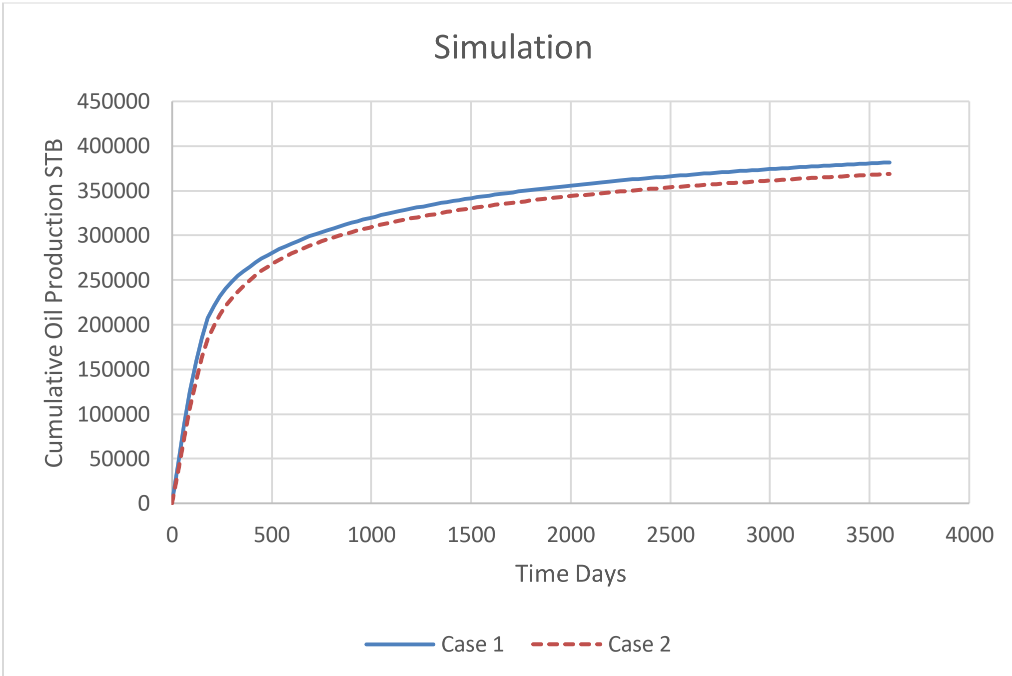

Figure 4.54: Numerical simulation cumulative oil production Case 1, Case 2

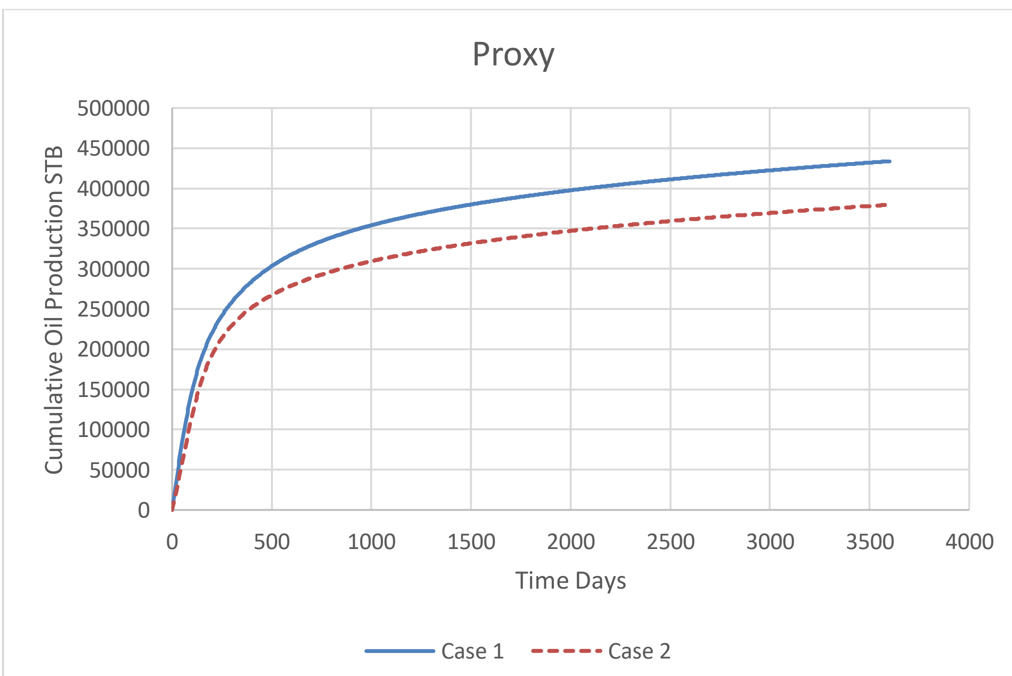

Figure 4.55: Proxy model cumulative oil production Case 1, Case 2

4.5 Inter-well Numerical Simulation Model (INSIM)

(Zhao et al. 2015) (Zhao, Kang, and Exploration 2016b) introduced an interwell numerical simulation model (INSIM) to evaluate reservoir performance under waterflooding process. The reservoir was treated as a coarse model having a number of control units connecting the wells, where each unit consists of two parameters: transmissibility and control pore volume, which are considered as the model parameters. By solving the mass material balance and front tracking equations for the control units, the interwell fluid rates and saturations are obtained so that phase producing rates can be predicted. The model parameters are estimated by history matching from the data available. INSIM model has the capability to estimate the oil and water rates, therefore history-match water-cut data which makes the model applicable for waterflodding optimization process.

Before the simulation process starts, it is necessary to specify the connections between wells in INSIM model. A connection radius for the wells inside the reservoir is considered based on the average well-spacing in the reservoir or the smallest distance between two wells, simply taking two or three times the selected parameter as the connection radius. For a specific well, any other well falls inside the connection window will be considered for a connection, and if it is out of the connection radius there will be no connection between this well and the subject well.

After determining the connections, the model parameters discussed earlier are defined at time zero. In order to start the history matching process, the initial guess of the parameters must be generated taking in consideration the available geological information. During the history matching process the values of these parameters are adjusted. The optimization parameters for the history matching process are the phase rate or water cut data at the producers, which are used as the objective function target.

INSIM model consider only the two-phase flow of oil and water, and solve the total mass balance for the two phases neglecting capillary pressure and gravity effect. Similar to IMPES (Implicit Pressure Explicit Saturation), in the pressure equation all the terms which depend on pressure and/or water saturation will be evaluated at the previous time step instead of the current step. After solving the pressure equations of all the wells implicitly, all the wells' pressures are obtained. Based on the well pressure, all the wells connected to it are considered to determine which well represents an upstream of the subject well. Buckley-Leveret theory and the fractional flow equation are applied to calculate the water saturation at the subject well due to the flow from the upstream connected wells.

In this work the INSIM model methodology was reproduced and compared to the proposed Proxy model and the traditional numerical reservoir simulation. Another version of INSIM, referred to as INSIM 2, was generated simply by altering the objective function of the model to have the different phase's production and the injection rates as the optimization parameters for the history matching process. The figures from Figure 4.56 to Figure 4.58 show the comparison between the Numerical reservoir simulation, proposed Proxy model and the two versions of INSIM model for Case 2. The parameters under comparison are the field oil production rate, field water production rate and field water injection rate.

In Figure 4.56 it is clear that both INSIM and INSIM 2 underestimate the oil production rate considerably, while, as shown previously, the proposed Proxy model gives a good estimate for the field oil rate.

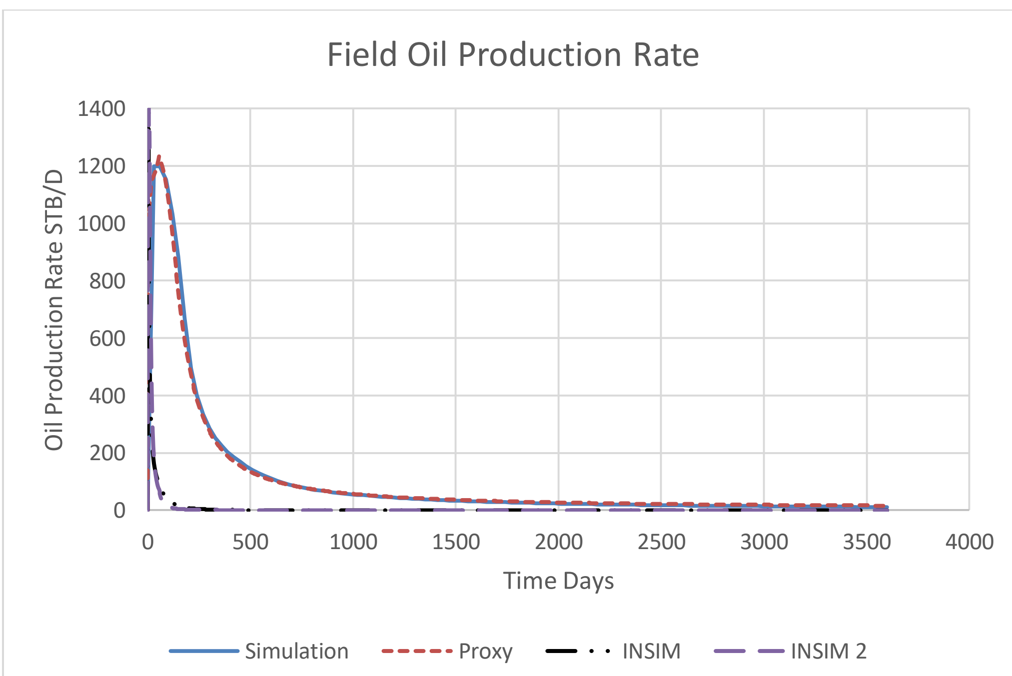

Figure 4.56: Field oil production rate comparison for Case 2

Looking at the field water production rate in Figure 4.57, INSIM model gave a poor estimate for the water production rate, while INSIM 2 gave a relatively better estimation. Both version of INSIM fell back behind the Proxy model when compared with the numerical reservoir simulation.

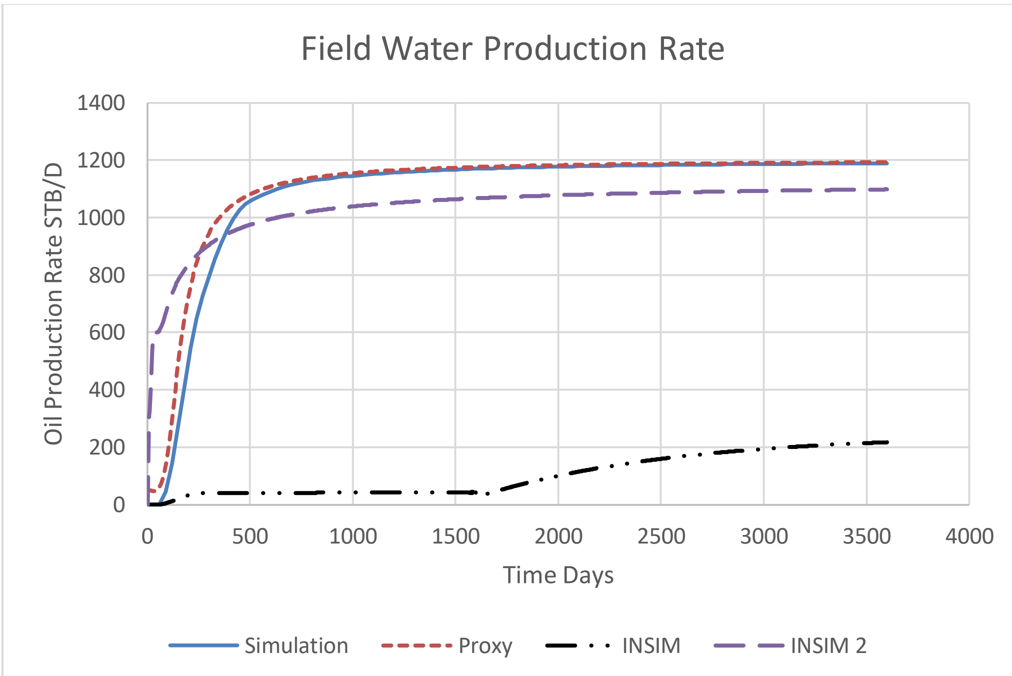

Figure 4.57: Field water production rate comparison for Case 2

Comparing the field water injection rate in Figure 4.58, all the models gave a reasonably good estimation compared to the numerical water saturation. However, INSIM and INSIM 2 did not show any effect with the maximum bottom-hole pressure constrain, as they give the maximum water injection rate for the whole duration of the simulation process.

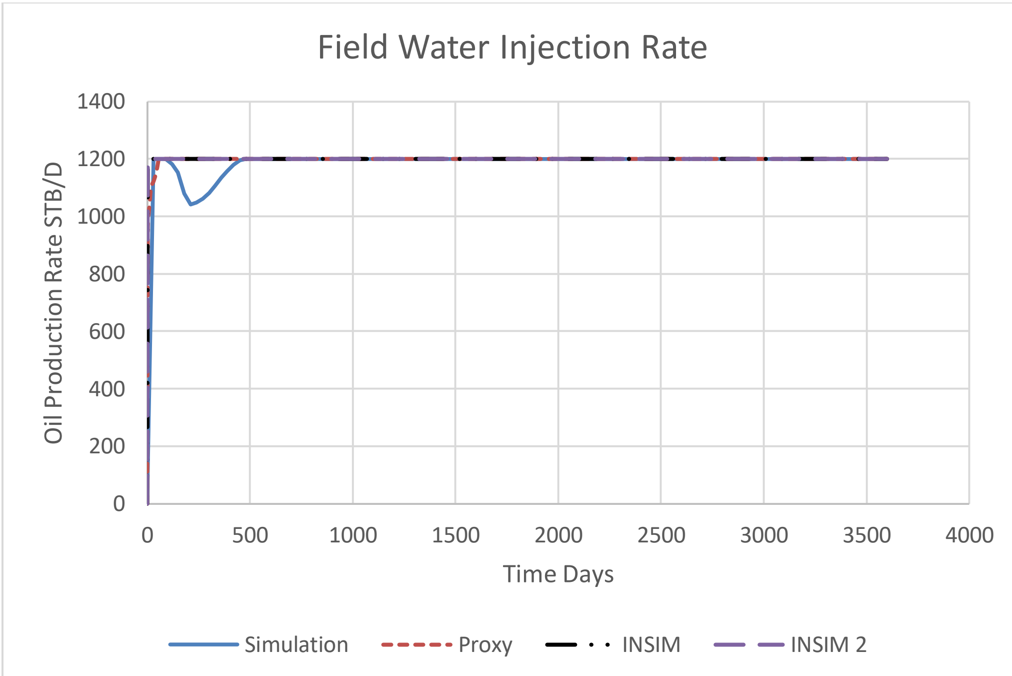

Figure 4.58: Field water injection rate comparison for Case 2

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

In this study a new, robust and inexpensive proxy model is developed for reservoir simulation. The proxy model is based on the diffusivity equation and incorporate reservoir static and dynamic parameters represented in rock properties, fluid properties and operational conditions. The performance of the model was compared with numerical reservoir simulation in the areas of speed, accuracy and computational cost. The result is a simple model that describes the reservoir performance at any specified point in time with a good level of accuracy in a fraction of a second. It can therefore be used in probabilistic forecasting, history matching, sensitivity analysis and developmental optimization.

The following conclusions can be drawn from this study:

- The proposed model can solve the problems of single phase flow with high speed and a good level of accuracy in a few of a seconds. This was demonstrated in the single phase section of the results where the model easily estimated the well pressure throughout the whole simulation period.
- The model can cope with all the boundary conditions including constant head, flow and no flow boundaries, existing separately or coupled in the same reservoir case.
- The proposed model can switch easily between various operational conditions, from specified flow rate to constant bottom-hole pressure and back to the rate constraint dependent on which condition applies.
- Proxy model can capture the change in the value of the flow rate, and can matches the same trend as exhibited by the numerical reservoir simulation models.
- For the same reservoir properties, porosity, permeability distribution, fluid properties, etc., the model can simulate different number of wells and well locations without the need to train again. The optimized parameters of the model are function

of the reservoir and fluid properties rather than the number of wells and their locations.

- The two phase flow simulated with the proposed model gave a good estimation for the oil production rate, water production and the injection rate in all of the producers and injectors.
- The predicted field oil production rate, water production rate and water injection rate estimated using the proxy model were a close match to those calculated using numerical reservoir simulation. It can therefore be said that the proxy model can give a reliable evaluation for the reservoir performance at any point in time.
- For the developmental study, the proxy model showed the same pattern as was shown by numerical reservoir simulation in the comparison between two water flooding cases. This shows the applicability of the proposed model for developmental optimization, as the proposed model elects the same case scenario selected by the numerical reservoir simulation.
- The proposed Proxy model outperform both INSIM and the modified INSIM 2 model in estimating the total field oil production, field water production and field water injection rate.

5.2 Recommendations

- The proposed model should be expanded to include three phase flow. As the model is based on the diffusivity equation, it is possible to perform the derivations for a three phase flow (oil, water and gas).
- The two phase flow example presented did not include capillary pressure effect, therefore another problem should be tested where there is more complexity as capillary pressure effect.
- Different types of reservoir should be considered in order to generalize the applicability of the model to every reservoir type.
- More developmental studies, like well placement optimization, should be conducted in order to investigate all the capabilities of the proposed model.

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